On the Meaning of Mathematical Incommensurability.

October 8 – Nov 2, 2016

I've written this article only for intelligent people. If you are not intelligent you are going to lose you time here but, because non-intelligent people love very much to lose their time, reading this post could be an enjoyable experience to you as well; take care, in any case, you could find out that after all you're smarter than you had thought and that might be not so funny.

The writing is very unconventional and unorthodox. It can have inaccuracies and errors. It does not matter to me. What is important here is to think. Start thinking by yourself.

> *"It is possible, of course, to operate with figures mechanically, just as it is possible to speak like a parrot; but that hardly deserves the name of thought".*
> *(Gottlob Frege. "The Foundations of Arithmetic").*

Think about how human beings could have started to measure linear lengths and areas. I guess to measure a linear length for the first time it was necessary to make an abstraction. We needed to start from a referential segment measured from one point to another point, based on a primary referential number of quantity; In that sense, we can suppose we created our referential segment of length 1 based on our primary number 1, one step, one thumb, one elbow, or whatever other one that represented a linear limited distance. With that referential segment 1 we already can measure any linear distance.

Our primary segment 1 is of course an abstraction, we did not measure how many points or lines are inside of that segment, we simply accepted it was right because it was very practical to consider that distance as our referential unity to measure linear distances, our initial referential metric; But that abstraction we performed is not a total abstraction because of inside of that referential segment there is a central point which divides the segment in to two equal segments; that central point determines the inner and perfectly proportionated symmetry of our referential segment 1.

Later, we also could create a referential segment of length 2 based on our referential segment of length 1, simply by repeating (or extending) the segment 1 two consecutive times. We now have a larger referential segment which also is a very practical thing to measure large linear distances. The original inner symmetry of 1 is perfectly respected because we can divide the segment 2 into to two equal segments of length 1. Here we place the intermediate point – which is a zero point – between the two segments 1 instead of being placed in the middle of a segment 1.

Until now the symmetry is still perfect. The first problem arises when we want to combine the referential segments 1 and 2 to measure another distance because then the original symmetry is lost: the segment 2 is larger that the segment 1, and we cannot set our central point of symmetry in the middle of those segments to save the symmetry. But that disproportion can be saved by creating a new referential segment based again on our originary referential segment 1. So we can create the referential segment 3: The symmetry is saved again, now by setting the originary segment 1 (with its central point) in the middle of a right segment 1 and a left segment 1, naming the whole distance our new referential segment 3. That is the way I think we can imagine how prime numbers appeared in geometry.

Prime numbers appear each time that our originary referential symmetry is lost and only can be fixed by creating a new reference based on our originary referential unity.

[I started speaking about the fundaments of the geometry considering linear segments, but I think it could be said the same thinking about the fundaments of arithmetic. To me, numbers are not abstract entities, quantities as well represent symmetry or asymmetry because for us – as human beings existing in a material reality – being aware or not – any quantity exists in a specific space and so has a spatial distribution].

Continuing our invented but plausible story about the prehistory of maths, we also can imagine how we were able to measure areas for the first time. For that we necessary needed to create a referential area based on our referential segment 1. So we took our referential segment 1 and built a square. And we agreed that the space inside of that square has the value 1. One square area. It is also an abstraction because we didn't measure that inner space in any way but the central point of that square of area 1 and its inner symmetry is not any abstract convention, it is a concrete reality.

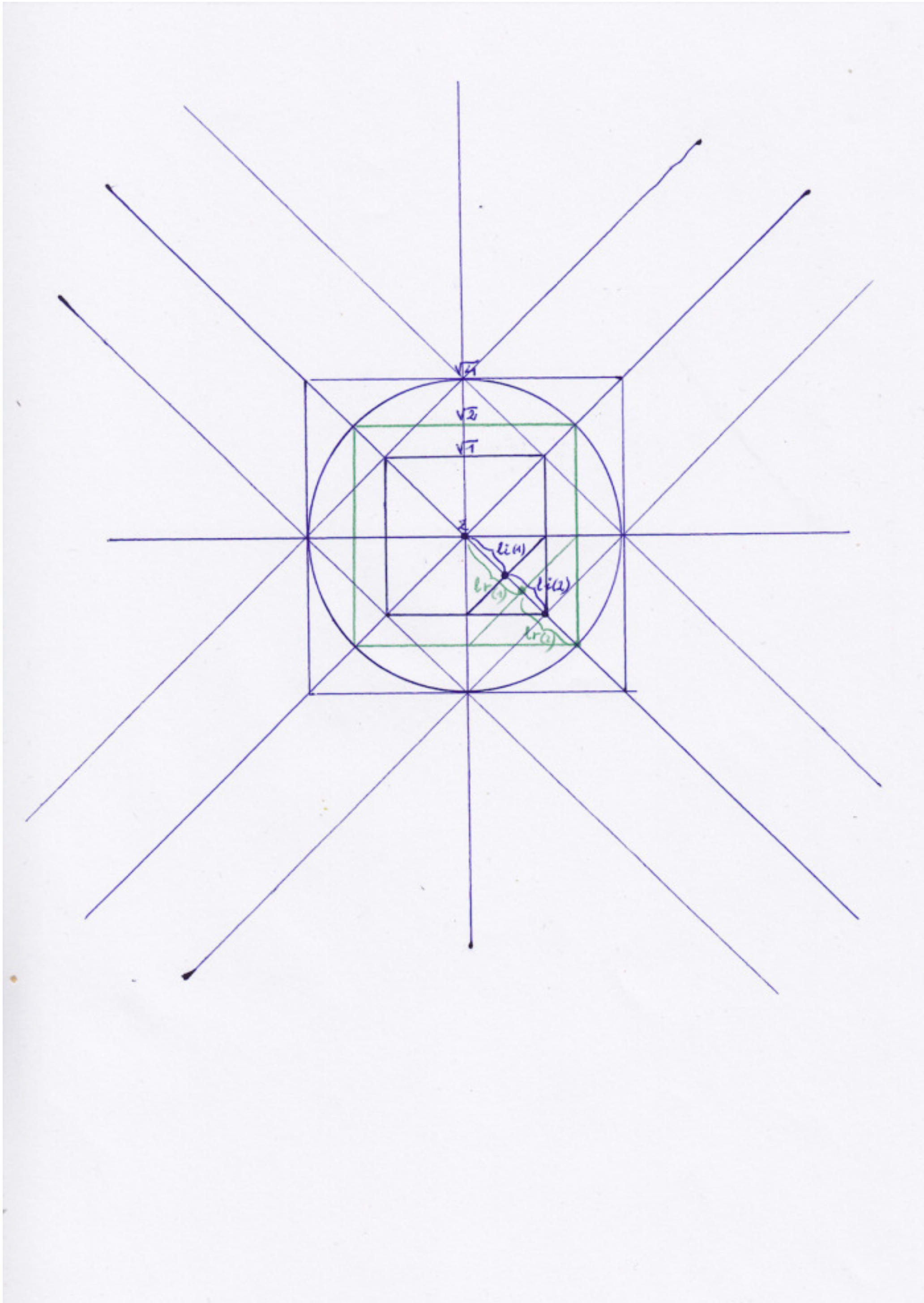
The next problem arouse when we traced the diagonal inside of our referential square of area 1. And that problem could not be saved in the same way that we solved the disproportion that appeared when it came to measuring linear distances, because now, with the diagonal, the disproportion appears inside of our referential square area 1 built on our originary referential segment 1. The disproportion appears inside of the originary unity itself. So, we cannot use the segment 1 to fix that disproportion created with the diagonal.

We cannot create a new referential segment with the distance of the diagonal based on our referential segment 1. And so we see horrified – as it is said ancient Greek people saw – that the hypotenuse of the equilateral triangle is irredeemably disproportionate with respect to the side 1 (the square root of 1) of our primary square area 1. The infinite decimals of incommensurable quantities appear, I think, because we are trying to compare two quantities and one of them is not related to (has not been derived from) our originary reference of metric.

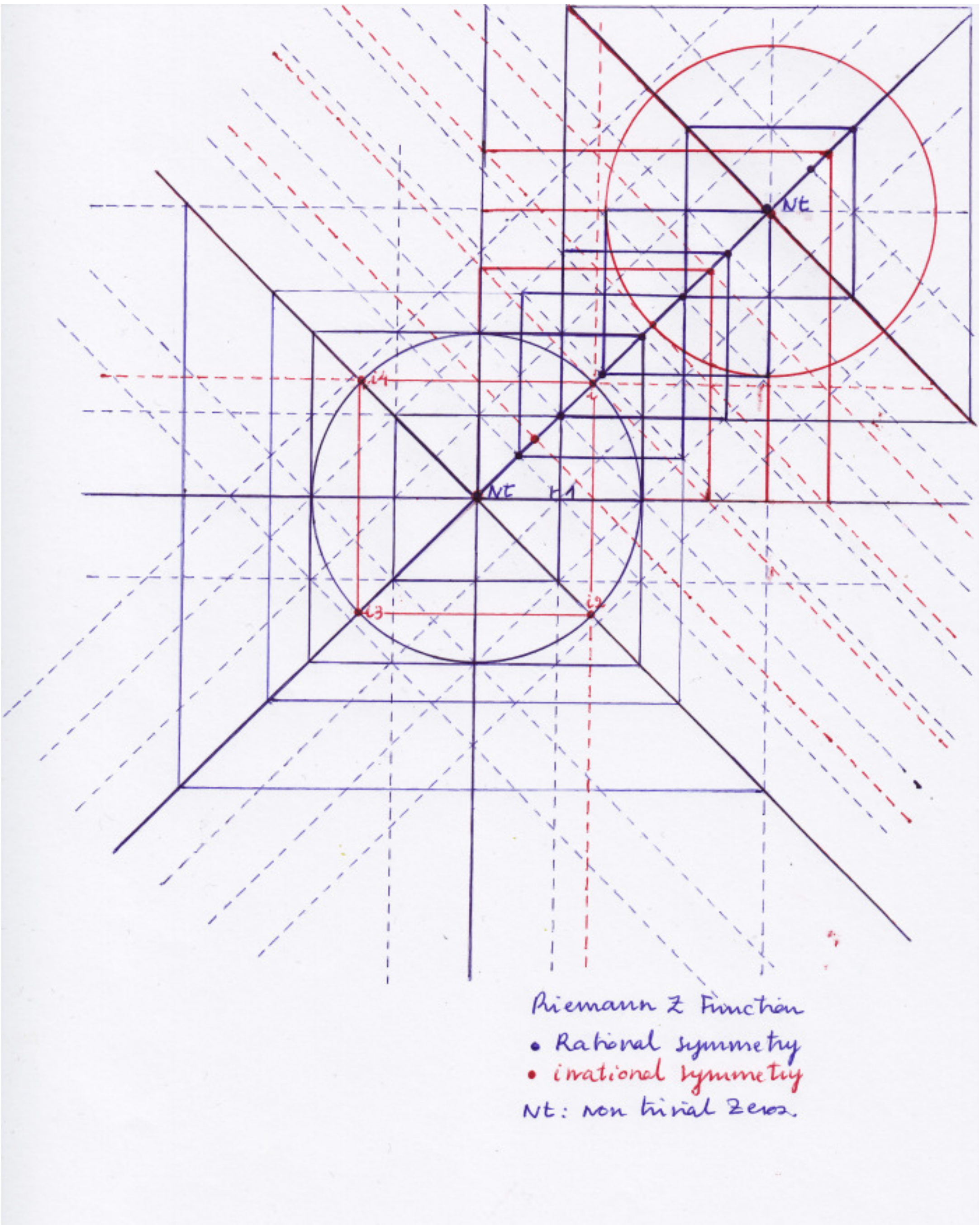
It occurs the same when we try to measure the area of the circle of radius 1 by using our referential square of side 1. We get the unsolvable disproportion between the perimeter and the diameter because they respond to different and disproportionate referential originary (or primary) metrics, referential originary (or primary) linear segments, referential originary (or primary) square areas: the rational and the "irrational" ones.

In this sense I think the circumference of radius 1 is a complex area formed by two kind of different squares that carry and are derived from two different kind of disproportionate originary references of metric that have two kind of different symmetries: the square of area 4, referenced to our "rational" (or irrational) referential square 1, and the square of area 2, related to an "irrational" (or rational) square of area 1"i". (I said "i" by expressing that it is incommensurable with respect to 1). The square 2 is inside of the perimeter of the circumference touching with its four corners the circle of radius 1, and the square 4 is outside of that perimeter touching with its four sides the circle of radius 1.

It is clearer when we divide the exterior square of area 4 in to four squares of area 1, and we divide the interior square of area 2 in to four squares of area 0,50, and then we set the central point of each of those different squares.



Measuring through any diagonal the distances from the central point of the circumference – the zero point to us – until the central point of those different squares we see that we need necessarily two different kind of referential intervals or referential segments of measuring, two different kind of referential metrics based on different originary referential segments, to reach the central points of symmetry of the squares of 0.25, 1 and 4, and the central points of symmetry of the squares 0,50 and 2. 0,25, 1 and 4 are ruled by the same referential segment, while 0,50 and 2 are ruled by another kind of referential segment.



I've used the term "intervals" because the resultant figures appear to be musical scales. If we repeat those two kind of referential intervals or segments, extending (or "projecting") them in a consecutive way through any the diagonal from the center of the circumference, we see that the two kind of disproportionate intervals concur or converge at a specific point of the diagonal: at the 7th and 5th intervals, the points which represent the two kind of referential centers of the two kind of disproportionate symmetries, converge.

I draw them using two different colours, red and blue. You can see there are 7 blue intervals with 8 blue points and 5 red intervals with 6 points. I'm considering the first and last points – the centers of the circumferences – as complex zeros formed at the same time by blue and red points.

Following the diagonal we can see:

- Zero blue an red points
- 1 blue point (blue interval 1)
- 1 red point (red interval 1)
- 2 blue point (blue interval 2)
- 2 red point (red interval 2)
- 3 blue point (blue interval 3)
- XXXXXX (no red point)
- 4 blue point (blue interval 4)
- 3 red point (red interval 3)
- 5 blue point (blue interval 5)
- 4 red point (red interval 4)
- 6 blue point (blue interval 6)
- XXXXXX (no red point)
- Zero blue and red points where the blue 7th interval and the red 6th interval converge at their respective end.

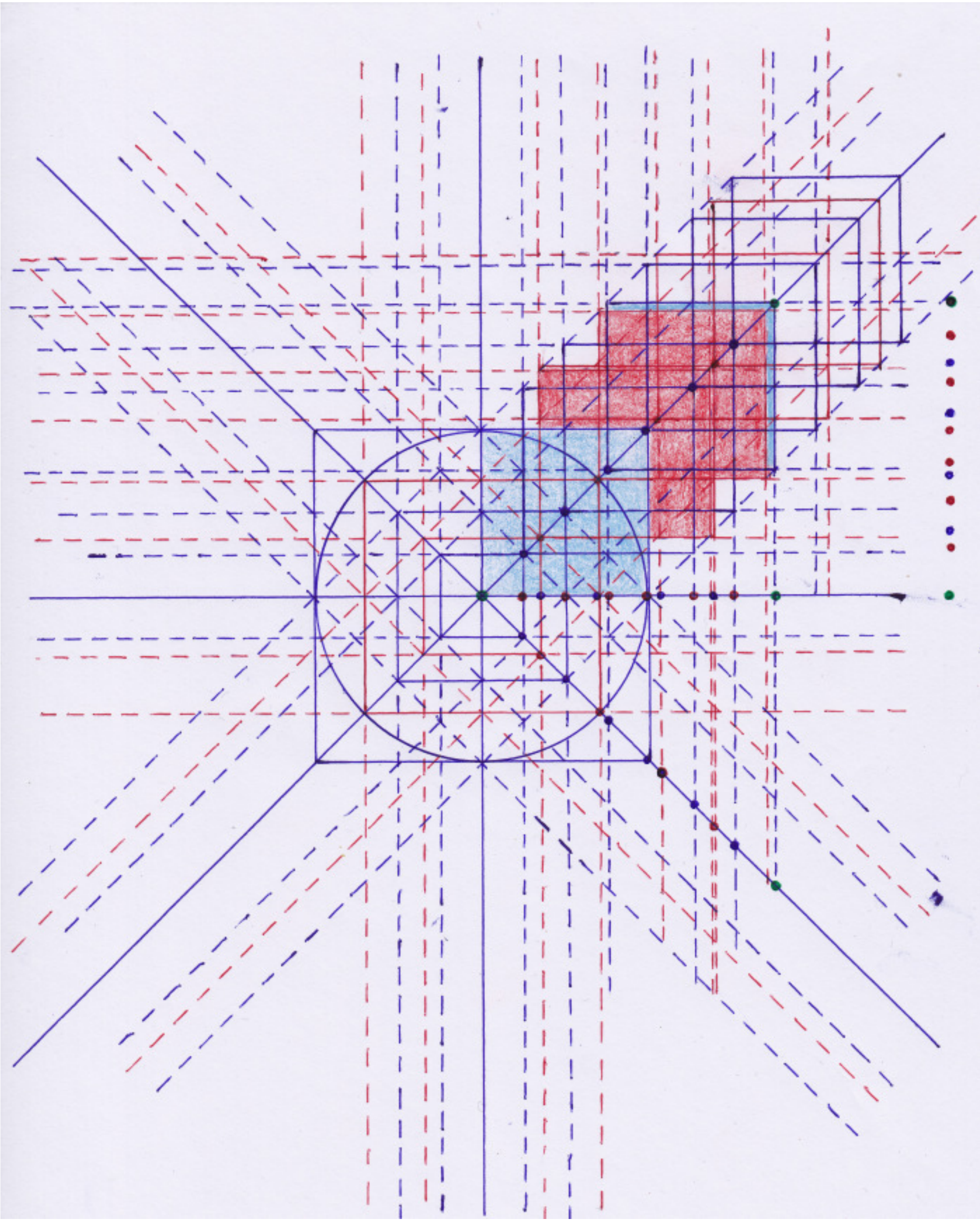
I think the intervals comprehended inside of the two rows marked with XXXX are what is known in musical terms as the "tritone". There, the periodical alternation between the two kinds of intervals is altered and it is perceived by our senses as something unexpectedly inharmonic. Until now, it has been considered as an irresolvable disharmony which appeared problematic because it seems to be born from the Nature itself; As a perfect Nature cannot be inharmonic, the tritone was historically prohibited during some time.

I consider each "Do" note would be in that sense a complex musical note (formed by the convergence of the rational and irrational referential symmetries) repeated periodically through the same scale.

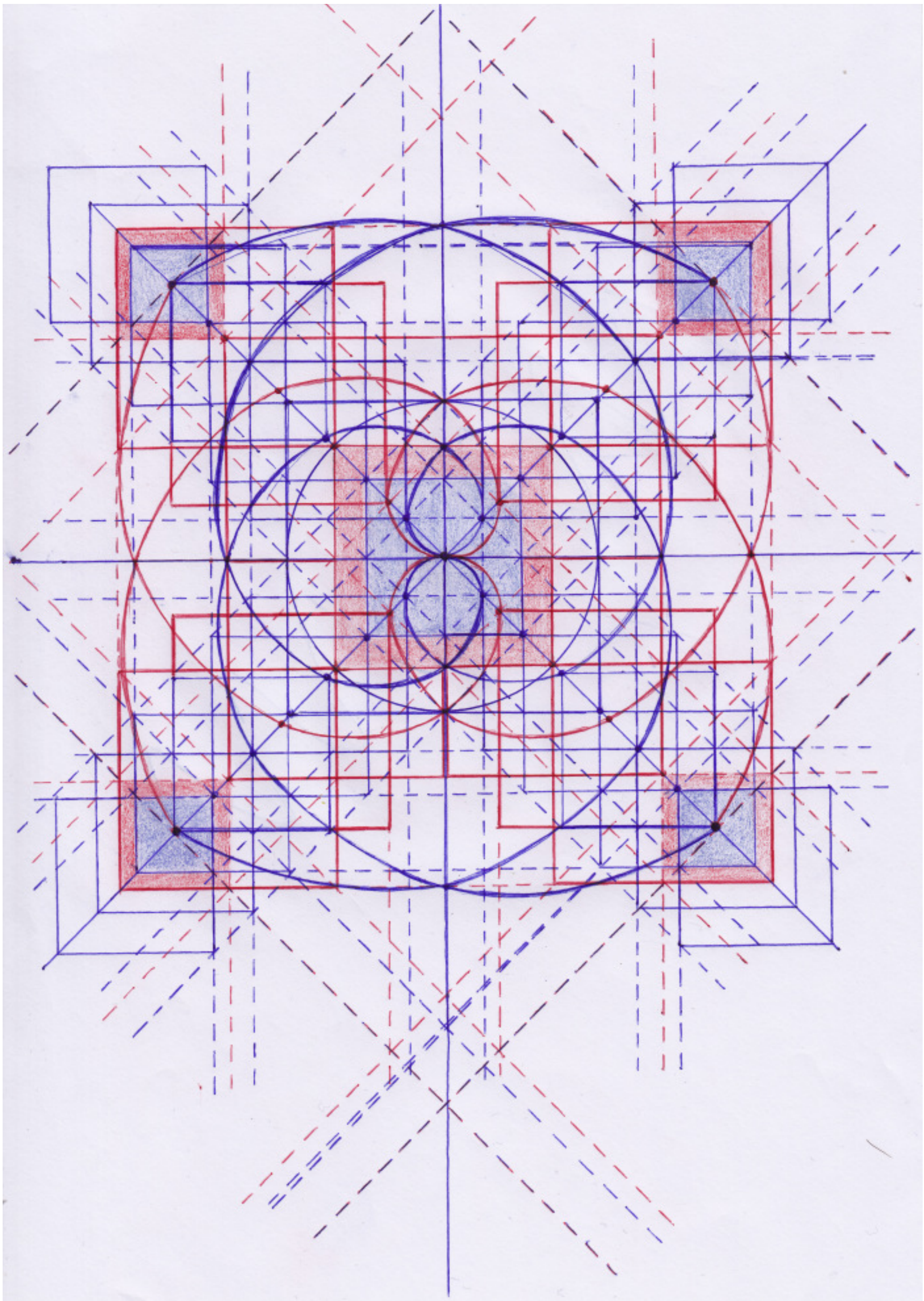
In this sense, I think the other non-convergent notes of a diagonal represent the only rational or the only irrational part of a complex note that only can be formed in a complete way by combining the notes of the scales of the Z diagonals and the notes of the scales of the XY coordinates. The intervals of the XY scales have an inverted order with respect to the intervals of the Z scales. (By saying here Z I mean the diagonals on a plane flat space).

We only need to seek where and how the intervals that carry the irrational and rational symmetries converge in a projective way. For example, on the X coordinate will take place the projective convergence of two intervals of the upper right Z and the bottom right Z coordinates. The whole space would represent then entangled metric scales that respond to the different kind of rational and irrational symmetries.

(About music and irrationality I think you could be interested in reading the beautiful article of Peter Pesic, published by the University of Chicago Press, *"Hearing the Irrational. Music and the Development of the Modern Concept of Number"*, which also clarifies the idea of number in ancient Greece).



On the other hand, when it comes to measuring the area of the circle, if we project three times the square area 1 through the diagonal following one of the mentioned intervals, from the zero point until the point placed where the perimeter is touched by one of the corners of the square area 2, we get a resultant area of value 1 + 1 + 1 = 3. We know the area of the circle of radius 1 is not 3, it is an irrational quantity, 3,14... with infinite decimals. But I think we should be able to measure the area of the circle without considering irrational magnitudes by taking account of the two kind of mentioned symmetries, considering the circumference as a complex area.



So, to measure the area of the circle by projecting our referential square 1 N consecutive times through the diagonal, we should project (displace) that square not only 3 times until the place where the perimeter is touched by the corner of the square 2 but also displacing the referential square – built on the radius of circle – until the next point which is related to the square 4 that touches with its sides the perimeter of the circumference.

On the above picture, the 0,14 (without infinite decimals) would be represented as the last blue stripe placed after the three consecutive red colored squares. I think at that extended point is not possible to accept the linear infinite decimals because at that point is where the irrational and rational referential intervals converge.

The circumference is a complex area that cannot be measured by using a unique and static referential square – attempts that historically were known as the "quadrature of the circle", which impossibility has been already demonstrated but not by using different referential square metrics – but considering the two kind of referential originary squares built on the two kind of originary referential segments that participate on its area.

The consequence of these ideas is that the use of a "mathematical fact" which is arithmetically correct as it is Pi, results mathematically wrong because of the a priori conceptual ideas that we have about the space we are working on are wrong and the instruments we use – the numbers or their algebraic representations – are not purely abstract entities existent in nowhere but concrete things that exist and are distributed in a specific space. Irrational magnitudes appear then as misunderstood mathematical aberrations. The Gauss' assertion about that arithmetic, and not geometry, is a priori and only its laws are necessary and true, appears invalid.

The mathematics developed since the XV century and all the large developments of the two last centuries have been made on the base of sacrifice our common sense in favor of utility. "Common sense", what appears to be "logic" to a sensible person, the Aristotelian common sense, was totally discredit to a trustable way to understand Nature since the Copernican revolution. The Aristotelian common sense had believed and imposed during more than fifteen centuries that the Sun orbits around the Earth. And since then, only mathematics is considered the only trustable way for understanding the whole Universe.

But when we currently say "mathematics" we are meaning "arithmetic" and "algebra". The geometry that is currently considered as true mathematics is an arithmetised geometry. And that arithmetization has been performed by the means of an algebrised arithmetic.

[When I started thinking about physics by myself looking for a mechanical explanation of anomalous cell divisions, I sent to many professors of physics of many Universities around the world some pictures and a text (with no mathematical calculus or equations but) with the explanation in a natural language of the atomic and solar system model that – after discussing with many unknown people and many friends. Between the few ones who answered my email, an emeritus professor kindly told me that my natural language was not comparable with the subtlety of the mathematical language he only understood when it came to physics.

Discussing later with other physicists I saw that no one was able to understand the simple pictures I was sending them; they frequently said that my pictures were artistic works with no mathematical value, and my ideas were philosophical – in the best of the cases – but not mathematics either. They said that if I wanted to discuss with them my conceptual ideas I had to develop them in a mathematical way. My conclusion was that the currently accepted atomic model had been developed in a blind way without having a visual idea about what the atomic structure is. Our physicists feel confortable inside of the mathematical abstractions that fundament the developed models regardless of whether they are "logically" coherent or they are not.

Actually, I could not have conceptual discussions with physicists because at some point, when they were not able to explain the conceptual inconsistencies and contractions of their models – for example when they had to explain coherently – outside of the mathematical formulas – the idea of virtual particles of the atomic model, or what a Majorana antiparticle is, or what the vibrating emptiness of an empty space is when it comes to explaining the Higgs mechanism, etc – they told me that things are in that way and end point.

So, I decided to try to provide to my models a mathematical language and I started thinking by myself about mathematics from the beginning, as I have made with physics, without having any previous idea on the matter.

And the first thing I found was that infinite decimals seemed a non-acceptable solution to irrational magnitudes, which to me seemed to have remained misunderstood until now. And I tried to understand by myself what irrational magnitudes are and how and where and when they can be related to rational magnitudes.

When I found what to me is a reasonable explanation, I saw, to my surprise, that mathematicians did not understand the pictures I sent them and the simple language I was using when, as I did above, speaking about originary referential segments or the originary referential square 1. I could understand that they were not used to thinking in those terms but I thought that the pictures, which were strictly geometric, would be perfectly understood.

What was going on, mathematicians do not understand geometry? I thought. One person even told me that the geometrical pictures were no so exact like equations, as if geometry were not true mathematics and only arithmetic and symbolic algebra were. It was very shocking to me to verify that they – current mathematicians – actually think in that way. The only geometry that is considered true mathematics is the arithmetised geometry which, in turn, has been previously algebrised. It explains that current mathematicians find so difficult to understand the Euclid's books. (See *the "5. First Interlude: Surd Numbers and Alogoi Magnitudes"* section of the David Fowler's article *"An Invitation to Read Book X of Euclid's Elements"*).

So the conceptual discussions were also impossible with mathematicians because currently geometrical diagrams are consider only as heuristic tools that does not probe anything, many (I'd want to say not all of them) mathematicians are used to working with totally abstract mathematical objects that they can operate with instead of thinking naturally in conceptual terms, and also because what I was telling them questioned directly the way that arithmetic has been used during the five last centuries. It implied that an arithmetically correct result could be mathematically wrong if the a priori idea we have – for example, to assume we are working on a unique flat plane that represents our flat space instead of being aware that we are working with multiple rotational planes overlapped like superposed layers on that flat space – is conceptually wrong.

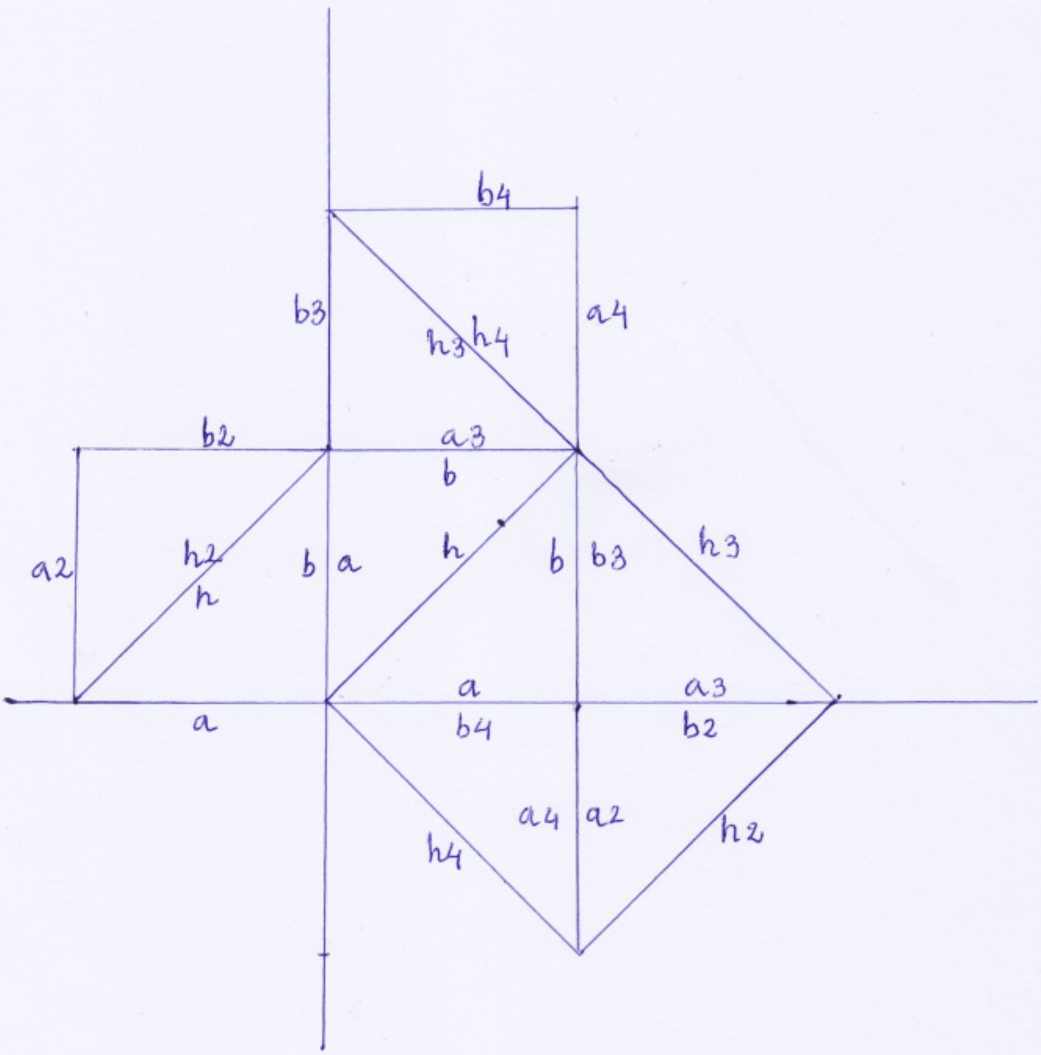
To consider this only idea seems to produce a violent reaction in some people that have internalized arithmetic in a very personal and intimate way as an absolutely trustable fundament of their worldview and personal believes. I remember discussing with someone that, instead of thinking about and trying to discuss conceptually the ideas I had told him about mathematical irrationality, simply performed a probability calculus in the sense that he considered mathematically impossible that so many brilliant people since the ancient Greece had been wrong while I, without even knowing mathematics and mistaking elemental terms as "circle" and "circumference", could be right.

In any case, I think the main problem behind those discussions is that the abstract idea of number has been highly formalized through arithmetic and algebra during the last two centuries since some mathematicians or logicians started thinking about the fundaments of arithmetic. But currently there is no a logical fundament of geometry outside of arithmetic, and the logical foundation of the number has been made only from an abstract – arithmetical and algebraic – point of view without considering that numbers are not abstractly ideal entities but that any number represents symmetry.

I think if arithmetic gives a conceptually unacceptable result we should still think about it and to review it instead of accepting it as a dogmatic true. So I think it should be necessary to reconsider why infinite decimals appear in a linear limited segment. Maybe it is because we are using our arithmetic in an incomplete way without considering all the hidden variables, that we are not taking account about all the circumstances that concur on the reality we are working with, or we are not respecting the rules we have previously assumed as it is that we cannot compare directly things of different nature, or all these things at the same time. What we cannot make is to scarify our reason for the sake of instrumentality. Rationality should be unrenounceable. Mathematics should mainly to think and to understand, not to operate. But it seems that when it comes to thinking we are limited by and classified in the realm of philosophy].

I tis told that incommensurability appeared for the fist time in the Pythagorean theorem. Here there are several conceptual inconsistencies. First, if $a^2+b^2=c^2$, it is logically incoherent that a^n+b^n Not = c^n (when n is higher than 2). Second, is incoherent that the area resultant from $a^2 + b^2$ is equal to c^2, because c^2 is built on a segment (the side of the square, the "root square") that is not commensurable, it cannot be measured by using the referential metrics that we use to measure the sides of the squares a^2 or b^2. How is it possible that those areas are equal if the sides of the squares cannot be compared without getting an incommensurable – "irrational", with infinite decimals although it is a limited linear segment – magnitude?

The area c^2 coincides with the areas $a^2+ b^2$ because the external elements of the squares a and b (the sides of those squares) are equivalent to the length of the internal elements of the square c^2 (the two hypotenuses placed inside of the square c^2; and the internal elements of the squares a^2 and b^2 (the two hypotenuses – four in total – of the squares a^2 and b^2) are equivalent to the external elements of the square c^2 (the sides of the square c^2). The sides of the squares and the hypotenuses of those squares carry the two kind of different originary symmetries, the rational and the irrational ones. (I say "symmetries" but it could be said the two kind of referential and originary metric magnitudes).



In this sense, any square a^2, b^2 or c^2, are complex areas that carry internally and externally the two kind of disproportionate referential metric symmetries. a^2+b^2 is Not equal to c^2, those square areas are coincident or equivalent but their inner and outer symmetries are different.

Square areas are not abstract entities that can be managed operatively in an abstract way by using arithmetically purely abstract symbols. Their symmetries are real and concrete and are distributed in a different although equivalent way.

If mathematics have been developed in such an abstract and instrumental way during the last centuries, and currently there is not a logical fundament of geometry outside of arithmetic, I think it is because of the lack of comprehension of irrationality. This lack of comprehension also would explain why the discovering of "imaginary numbers" took place so late as mathematical objects suitable to be used arithmetically or algebraically but without understanding why complex numbers appear and their conceptual meaning. Without having conceptual clarity the way gets slow and tortuous and the necessary efforts to advance become heroics and isolated and appear to be created or invented (or discovered in the best of the cases) by the enigmatic and obscure processes of portentous imaginations.

I think that understanding irrationality in the way I explained above clarifies conceptually what complex numbers are and lets to understand geometrically the problem of the insolubility of quintic and further equations with radicals, which was treated by Galois in an abstract way. I think it is related to the same problem of the disharmony that appears in the musical scales with the named "tritone".

I also think the complex zeros that appear when the mentioned intervals converge (saving the periodical disproportion between the two kind of originary symmetries) are the same thing than the named Riemann's "non trivial" or "relevant" Zeros when it comes to looking for the periodicity of the appearance of prime numbers.

I consider that what I above called referential metric intervals, or referential originary kind of symmetries, are similar to what Bernard Riemann considered as a "quanta" of varieties (or manifolds), and also the notion of "multiply extended magnitudes", in an abstract way, in his conference about the "Hypotheses which lie at the bases of Geometry"; And also i think those ideas are a similar to the "gauges" considered by Hermann Weyl as different distance scales.

(About Weyl's Gauge Symmetries and Gauge Theories – which have been key in the development of quantum mechanics – you can read the Chapter 5 of *"Not Even Wrong" by Peter Woi*t).

I think both authors actually where speaking – each one from their limited perspective and creating new terms for manipulating in an instrumental way the same things – about different referential linear metrics carrying different kind OF immediately disproportionate symmetries. And they both made their developments in a blind way without understanding irrationality.

In this sense I suspect that the named "non Euclidean geometries" are using – without being aware, also in an abstract and instrumental way, through algebra and arithmetic – something similar to the different kind of referential magnitudes I mentioned above, their connections and their local transformations, and I think it is the root of its difference with the Euclidean geometries which only would use one kind of referential linear magnitude, the rational one.

I'm very curious about reading and researching in the books V and X of the Euclid's elements, which treat respectively about the theory of ratios and the concept of magnitude, and about irrationality.

These ideas related to the History of maths I mentioned above are clues that I will try to developed in a clearer and deeper way if I have time.

Another clue I'm going to developing when having time is related to a very interesting figure who is the German mathematician Gottlob Frege. Ignored during his live, he is currently considered the founder of the modern mathematical logic and the father of the analytical philosophy thanks to the works of Russell and Wittgenstein. Frege tried to derive the whole arithmetic from logic, that is to say, to reduce arithmetic to logic. He was very aware that mathematics "was inadequately supported.

> "*This entire impressive construction, he claimed, rested on shaky foundations. Mathematicians did not really understand what they were about, even at the most basic level. The problem was not a lack of understanding of the true nature of imaginary numbers, or of irrational numbers, or of fractional numbers, or of negative integers; the lack of understanding began with the natural numbers such as 1, 2, 3. Mathematicians, in Frege's view, could not explain the nature of the primary objects of their science".*

And that's why he tried to setting out the logical and philosophical foundations of arithmetic. (The quoted phrases of the above paragraph are taken from *"Frege, an introduction to the Founder of Modern Analytic Philosophy", by Sir Anthony Kenny*)·

(I will add other books I'm interested in at the end of the post in a bibliography section. I'll try to read all of them to improve the exposition with more and also more accurate and interesting data. Although making this exposition in an erudite and complete way would require working on it in a large and continued way spending a lot of time that I currently do not have. I hope I will be able to still make some improvements slowly).

I think it is not possible to deduce arithmetic from Logic – even in an abstract way – without previously understanding what irrational numbers are from a conceptual point of view.

Frege reviewed the concept of number and spoke about classes and extensions of quantities as a key point of his logical system. But his whole system become untenable because of the objection that Bertrand Russell made about it. I'm very curious to read more about the Frege system and the Russell's objection because I think if the system became inconsistent, Frege was very secure about his validity before getting the Russell's critic, its inconsistency could be related to the lack of comprehension of incommensurable magnitude sand how they are related to rational ones, the ignorance about the existence of at least two different kind of referential magnitudes that create periodical asymmetries. (Ideas that as I mentioned above I think were developed by Riemann, Lie, and Herman Weyl, using different terminologies and from limited perspective). Frege was not able to fix his logical system after the Russell's objection (the Russell's Paradox).

I think maybe there are still some things to say about the named "logicism" considering it under the light of a clearer comprehension of irrationality and considering numbers or referential metric magnitudes, linear extensions, and areas, from the point of view of symmetry. To me is evident that irrationality has not been yet understood at all.

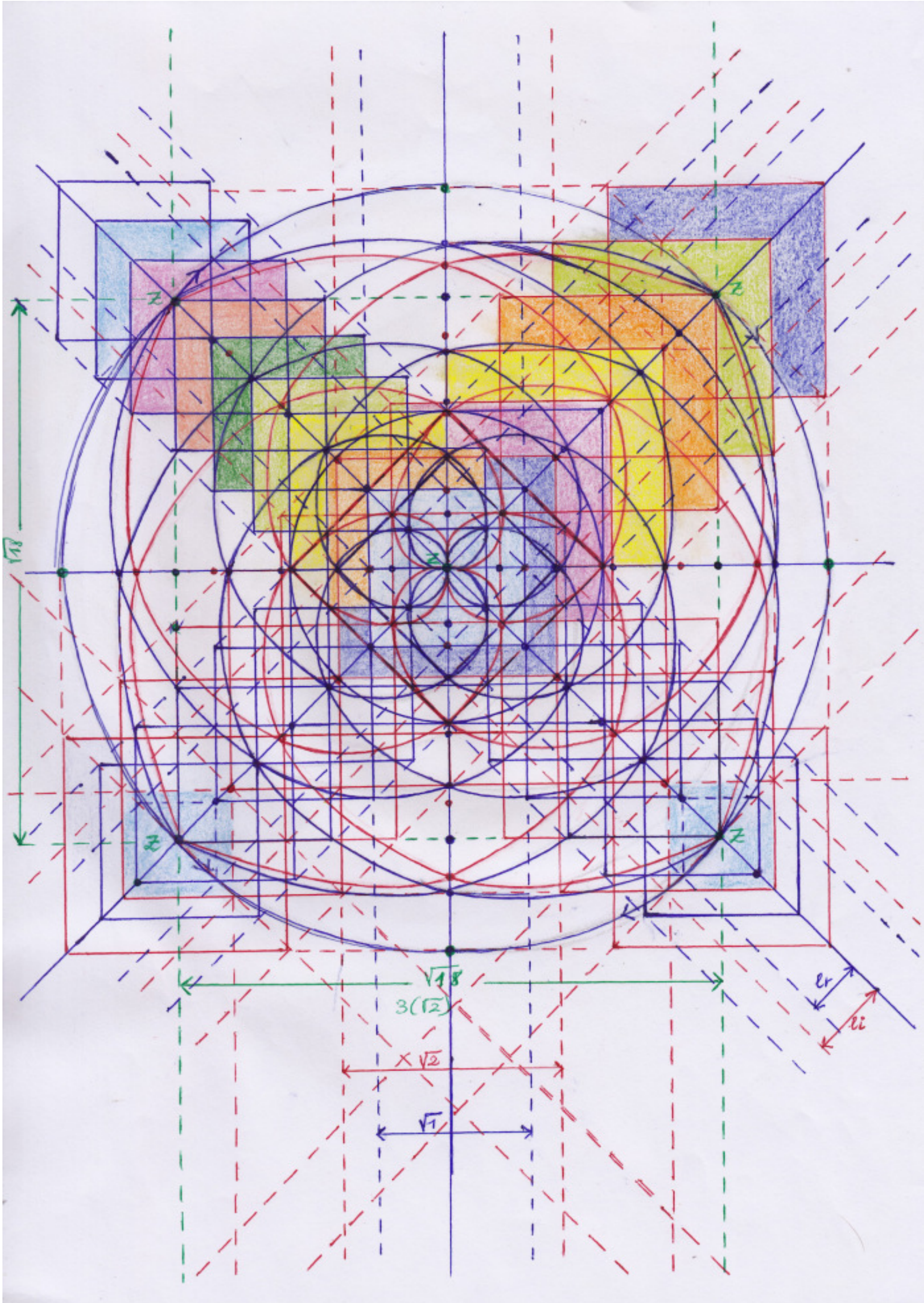
Thinking about modern mathematical logic in terms of "classes", if we think in primary-generatrix classes, as if they were sets, and the subclasses derivate from them we would need to think about the kind of symmetries those subclasses carry or represent externally (related to their perimeter) and internally (related to their areas), how those different symmetries are periodically disproportionate – cannot be immediately compared, connected or related – how they can be connected locally and periodically (forming a "continuum"), and how the equivalence of those symmetries can occur by comparing the exterior with the interior symmetries of those subclasses. The subclasses-square-areas are derivate extensions of the two primary classes (the referential square rational and irrational areas), which are derivate from the two primary linear classes (the rational and irrational referential segments, which are derivate from the number-quantity 1, the abstract "monad" without extension, in terms used by Leibniz.

On the other hand, coming back to the different kind of symmetries and their relation to the problem of quintic equations considered from a geometric point of view, I think the pictures below show where and when we can and cannot pass through one kind of symmetry to the another one making "local transformations" of symmetry. (To understand the pictures below as equations of n degree it is necessary to consider the square areas built on the rotatory axis.

When we speak in geometry about 2^2 we are not speaking about an abstract entity which we use as an arithmetic instrument for performing arithmetic operations without a specific meaning, we are speaking about a square area built on a segment which has a specific symmetry; here, that segments, the sides of the n squares are the segments traced from the zero point toward the different directions, which actually is the same segment displaced in a consecutive way toward the different directions for drawing the circle.

I would recommend you to draw the pictures by yourself starting from the beginning and to observe them by yourself; by making that you will be making a geometric analysis. That is a concrete analysis.

If you are not used to thinking in these geometrical terms without using abstract algebra and arithmetic, without having equations to clear, it won't be easy to you to understand these pictures by only looking at them here.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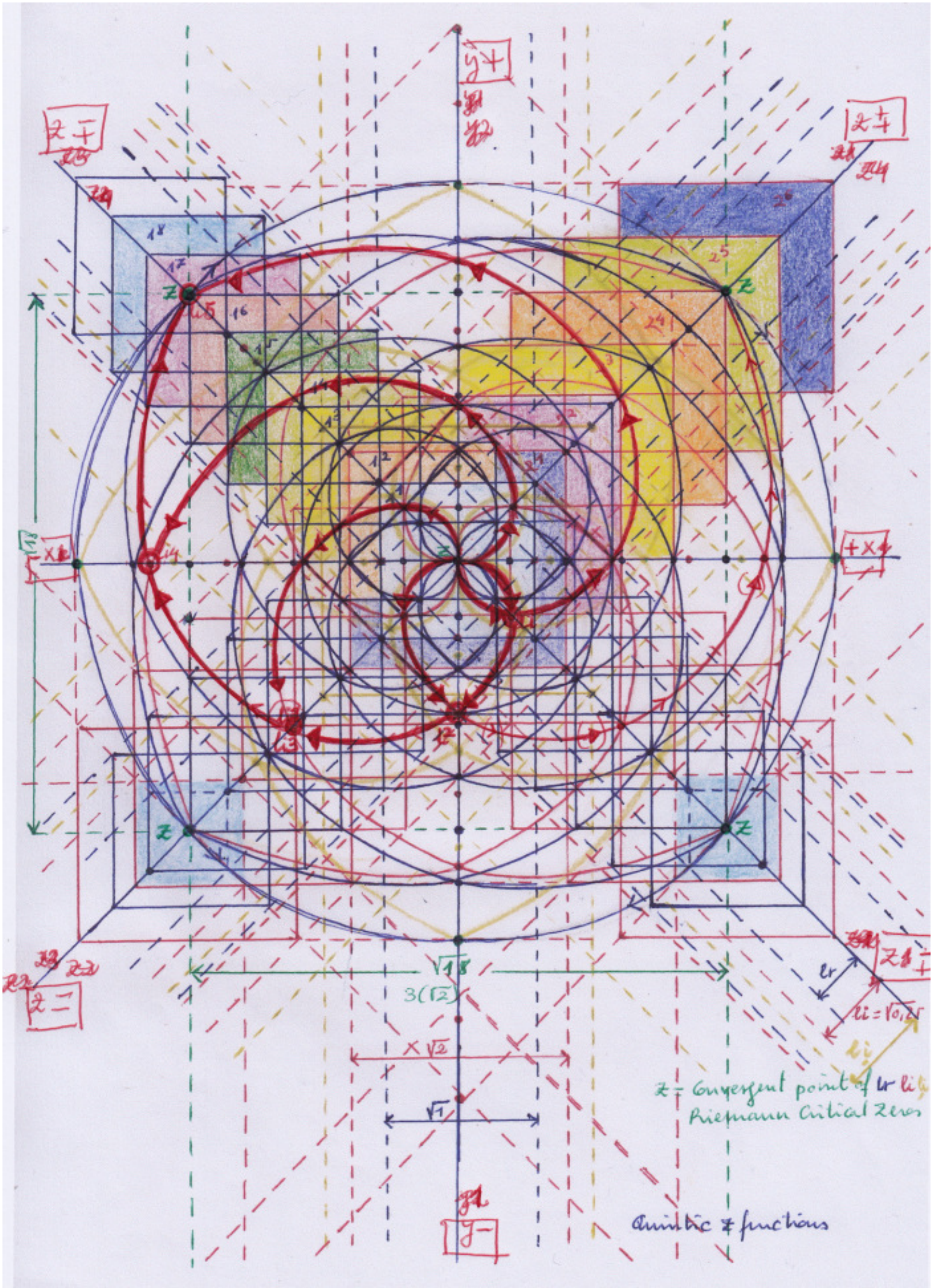
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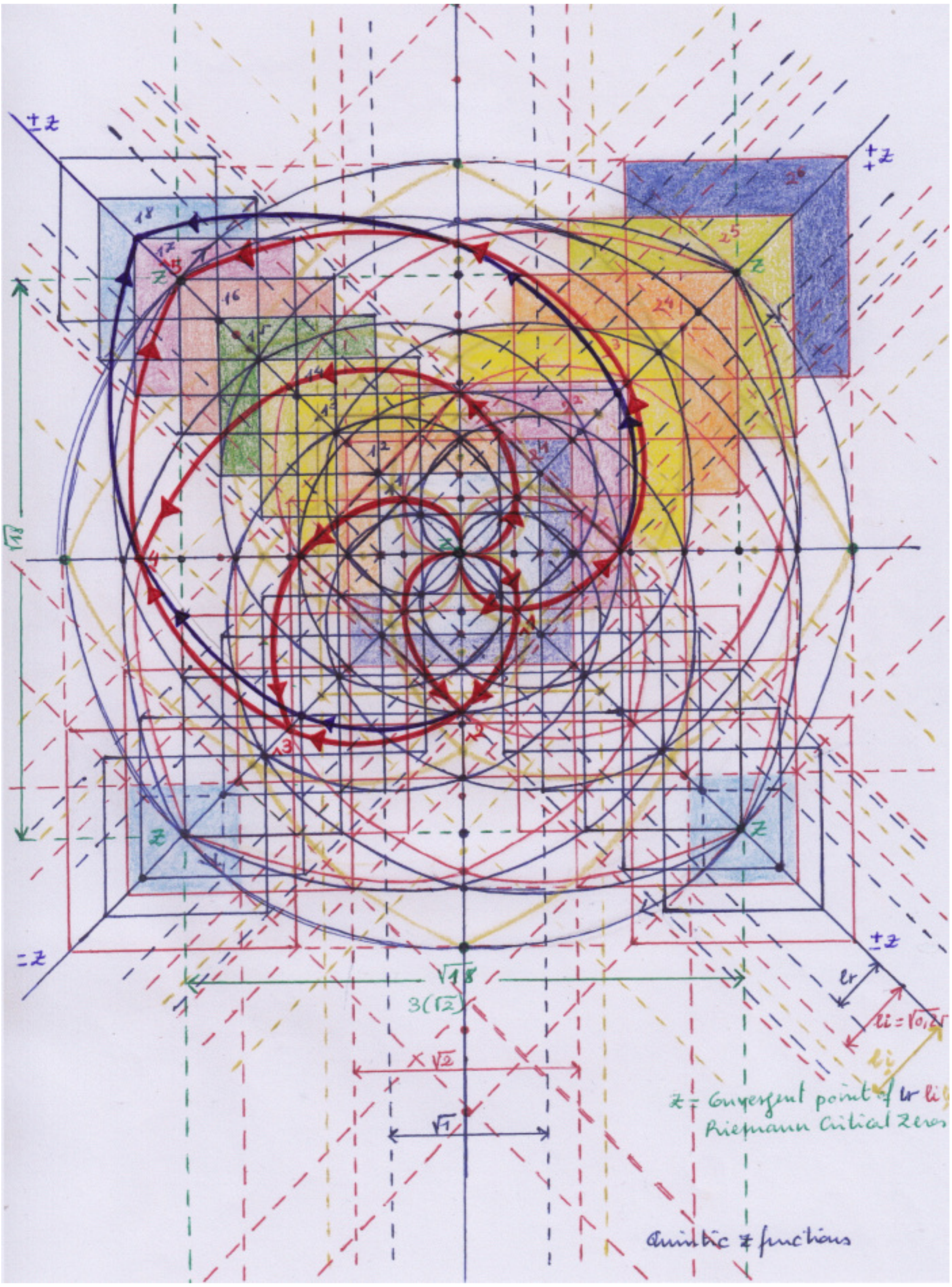
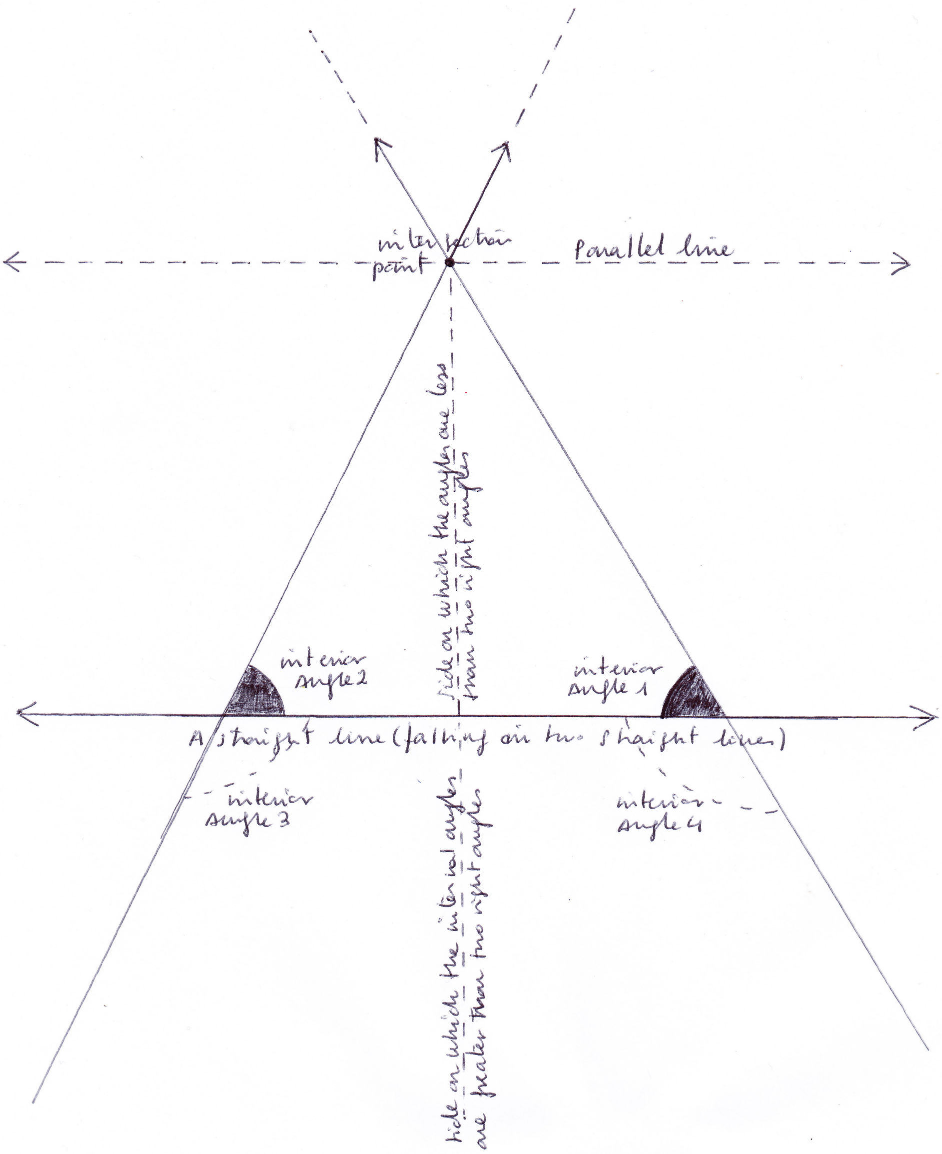


I said above "the rotatory" axis because I think that we cannot consider the Z diagonal as an independent axis on the flat space we are working on. The Z diagonal is the same Y or X axis displaced toward the right or left.

We have no problem by displacing the XY coordinates if we displace the whole flat plane, but if we want to preserve our original XY referential coordinates and introducing a new coordinate Z on the plane space, we need to be aware that we have superpose a new plane – the one represented by Z – on our working space, and that Z plane has rotated with respect to the initial position of our original plane related to XY. The actual effect is the same as if we had expanded our physical plane-space when we are considering the 4 Z coordinates from the point of view of XY, or as if we had contracted the expanded plane-space when we are considering the XY coordinates from Z.

The same can be said when we think about the orthogonal plane. Considering X like a horizontal plane and Y as a vertical plane, perpendicular to X, we have no problem because we are working with a unique quadratic and so rational reference. The problem appears when we displace (without been aware about that displacement, thinking that it is an independent orthogonal coordinate) Y toward the right or the left.

In the XIX century the so-called "Non-Euclidean geometry" appeared. It is known as non-Euclid because it is based on the refutation about the 5º Euclid's postulate of the "Elements": ″If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles″.



From the 5º postulate it can be deduced the so-called “Playfair’s axiom”: “There is at most one line that can be drawn parallel to another given one through an external point”.

Non-Euclidean geometries consider that it is possible to trace more than one parallel line upon that point. Could you imagine how?

Non-Euclidean geometries were initiated (“discovered” if you will) by Lobachevsky in an arithmetical and purely abstract way. Lobachevsky himself called his geometry “Imaginary geometry” because of it’s total abstraction. The 5º Euclid postulate is very simple and self evident but there was no arithmetical probe about it. Many mathematicians tried to find out and arithmetical demonstration; as none of them was able to fond it, Lobachevsky, in the XIX century, tried a different kind approach, to try to demonstrate that the 5º Euclid postulate was wrong.

As Euclid did not define in an exhaustive way the concept of straight line, Lobachevsky thought it was not contrary to the other Euclid postulates to consider that a curved line following a straight direction, a straight line on a curved plane and a curved space, can be considered a straight line as well.

Euclid had defined parallels as “straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction”. How can we represent geometrically, visually, then, those multiple parallels lines passing upon the same point on a curved space? Lobachevsky did not represent them, but later other mathematicians started to be able to represent non-Euclidean (curved) geometries, also named “hyperbolic”, for example on a sphere (The Riemann Geometry), or in a pseudo sphere (like a funnel).

The parallels must be parallel with respect to the first line but do not need to be parallel (equidistant) between them, so they can converge on a point (for example the pole of the sphere) in one of their sides and to diverge in their other side.

I think that even admitting that curved lines with straight directions are straight lines, for example the circle on a section of the sphere, it will be necessary to be aware that there will be two points of intersection, and that the parallel lines will not have the same kind of symmetry when they pass through the front side of the sphere with a concave (or convex) curvature and when they pass through the back side of the sphere with a convex /or concave) curvature.

And also in any case, it will be necessary to realize that the curved lines on the curved spaces, or the curved lines on a plane section of the curved spaces are always going to be measured with our quadratic referential areas and their totally right segments traced and existing on flat plane. Because we do not have curved referential segments based on our originary referential unity.

We could create a referential curved segment if we will to create a new way for measuring spaces and areas but then we should be clearly aware that we are doing that. In other case, when it comes to measuring curved lines on curved spaces, we will be always using at least two kind of quadratic and non-curved referential segments and referential square areas that carry our referential originary magnitudes, the rational and the irrational one, in the way I explained above.

In my view non-Euclidean geometries are actually non-Euclidean because the parallels they are working arithmetically with are placed on different planes, while the Euclidean parallels are on the same quadratic and rational plane.

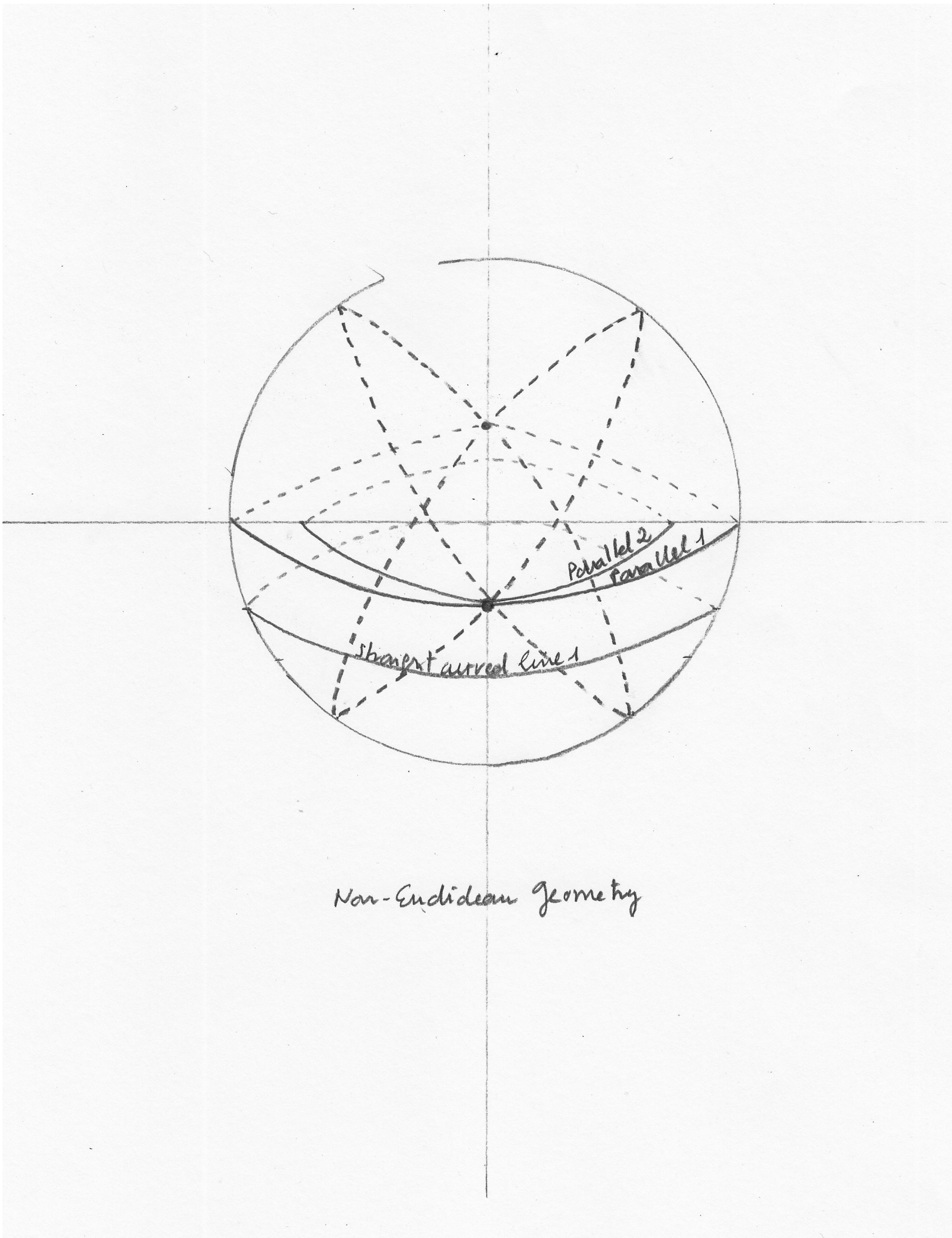
As I said above, I think each time we trace a circle (to create a circumference or a sphere), we create a complex area (or volume) where concur different referential metrics expressed on different superposed planes – like if they were different layers – on a same flat space. The Z (orthogonal or not) coordinate is not a simple tool we can use to managing our space symbolically. Since the moment we are using Y with Z or Z with X, we have created two different planes related to different referential and disproportionate metrics, the rational quadratic plane of X and Y, and the irrational quadratic plane of Z. It is the same issue that I mentioned when I spoke about irrationality.

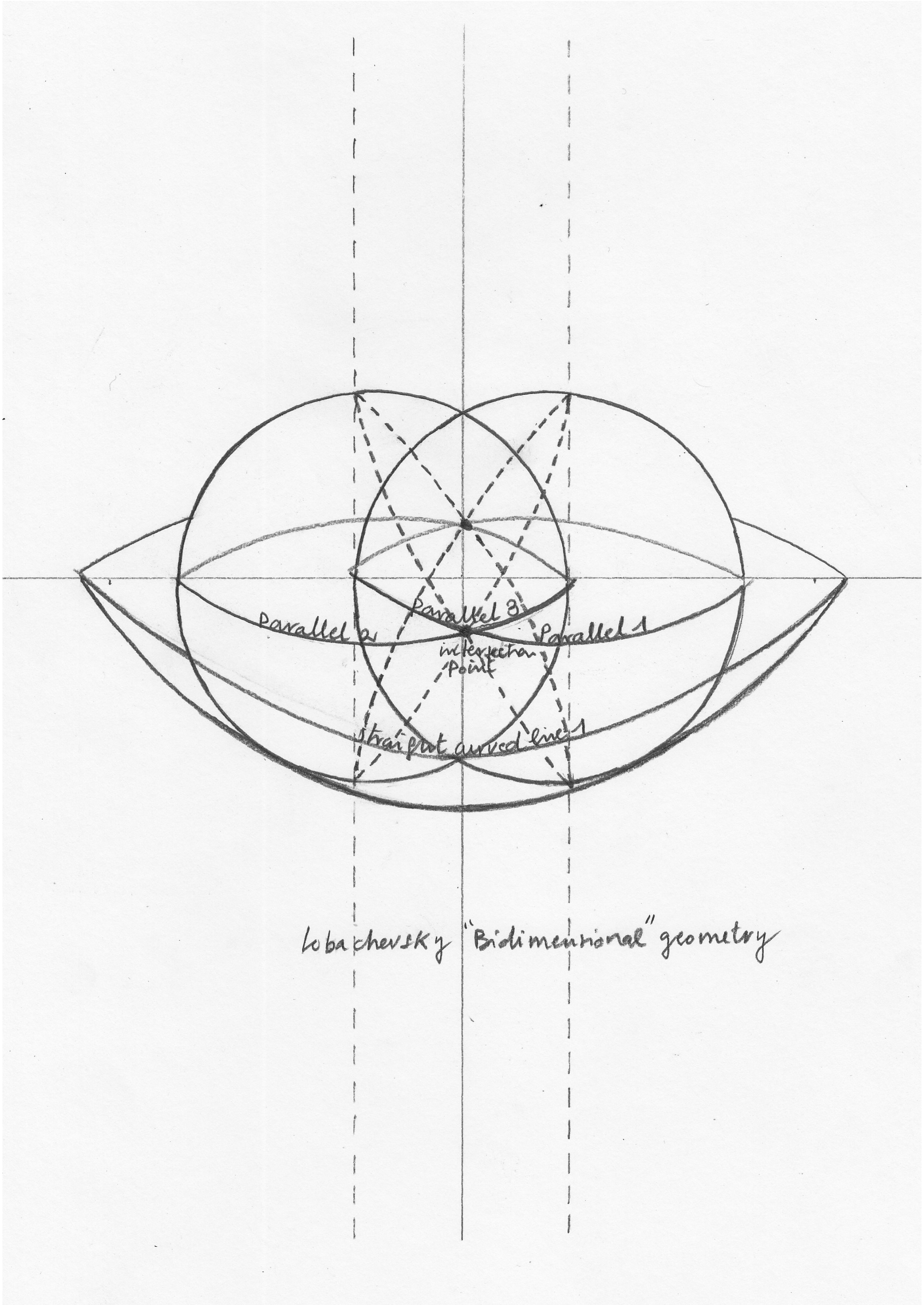
The effect of displacing Y toward Z without displacing at the same time the quadratic YX coordinates, without rotating the whole initial XY plane, implies to modify our primary referential metric, and the actual effect is equivalent to expand the physical space we are working on; and the effect of displacing Z toward X without displacing YX at the same time, is equivalent to contract the physical space we are working on.

It seems, it will be interesting to research about it, that Greek mathematicians were not aware either they were working with complex areas and different planes on the same space related to at least two kind of referential metric magnitudes periodically disproportionate. In this sense I would say the Euclidean postulates remained incomplete because of the lack of comprehension of irrationality.

So I think the "non-Euclidean" geometries are non-Euclidean because they do not share the Euclidean definition of parallels without being aware of it. I think our mathematicians have not yet understood irrationality and are not aware that their curved parallels are placed on different planes on a same space. It always happens when it comes to working with curves. You can consider curved lines as straight lines if you will because they follow a straight direction, but curved lines necessarily will pass through different planes – the rational and the irrational ones – of the complex – flat or non-flat – space.

On the pictures below I've drawn what I think would be the Lobachevsky "imaginary" two-dimensional geometry:


Parallel 2
Parallel 1
Straight curved line 1
Non-Euclidean Geometry



We also can introduce a motion on the intersected spheres, creating a periodical expansion and contraction on each field, which would represent different moments of the atomic model I'm going to speak about now.

In that case, the curved parallel lines will be only periodically parallels and their behaviour will be different depending on the phases of their periodical variation, if they expand or contract at the same time or at different time, and their periodical synchronization and desynchronization.

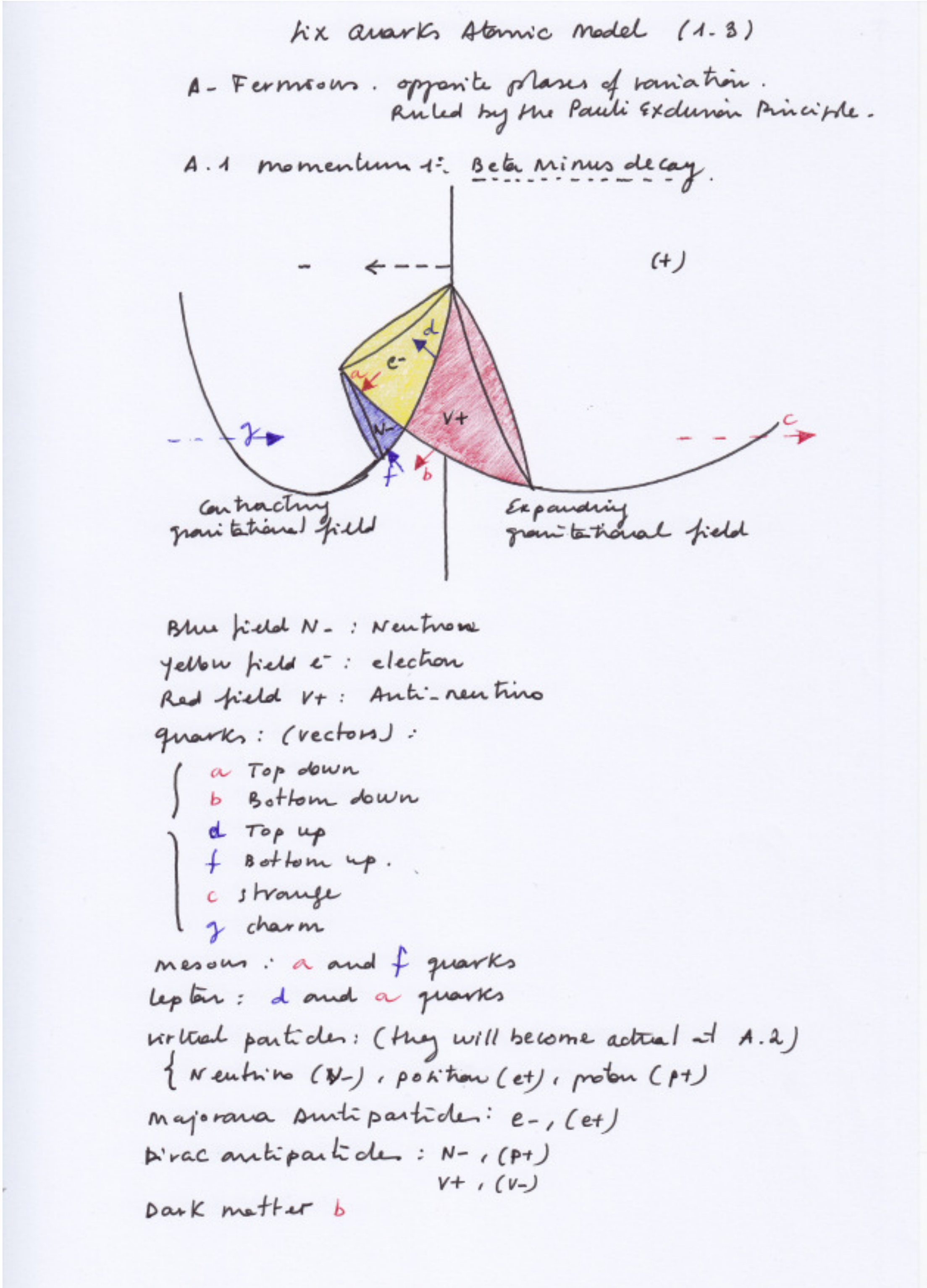
Related to the idea of the periodical expansion and contraction, I also think that our physicists have tried to measure our Nature from their static and quadratic references what have made things more and more complicated. Think for example that the gravitational field we are orbiting expanded and contracted periodically.

It has not been detected yet, but if it were it would have many implications because instead of considering the apparent but actually inexistent orbital ellipse (the orbital ellipse does not exist, it is only a figure we construct when we see the planetary motion) we should had been considering the circular and periodically expanding and contracting gravitational field, which we think is existent although it is invisible.

That periodical expansion and contraction would cause the planetary motions and their periodical acceleration and deceleration. The "Big bang" of our universe would be only a moment of the periodical variation of two intersected fields varying with the same phase, contracting at the same time. When those fields expand at the same time it appears the opposite phenomenon, a "big silence".

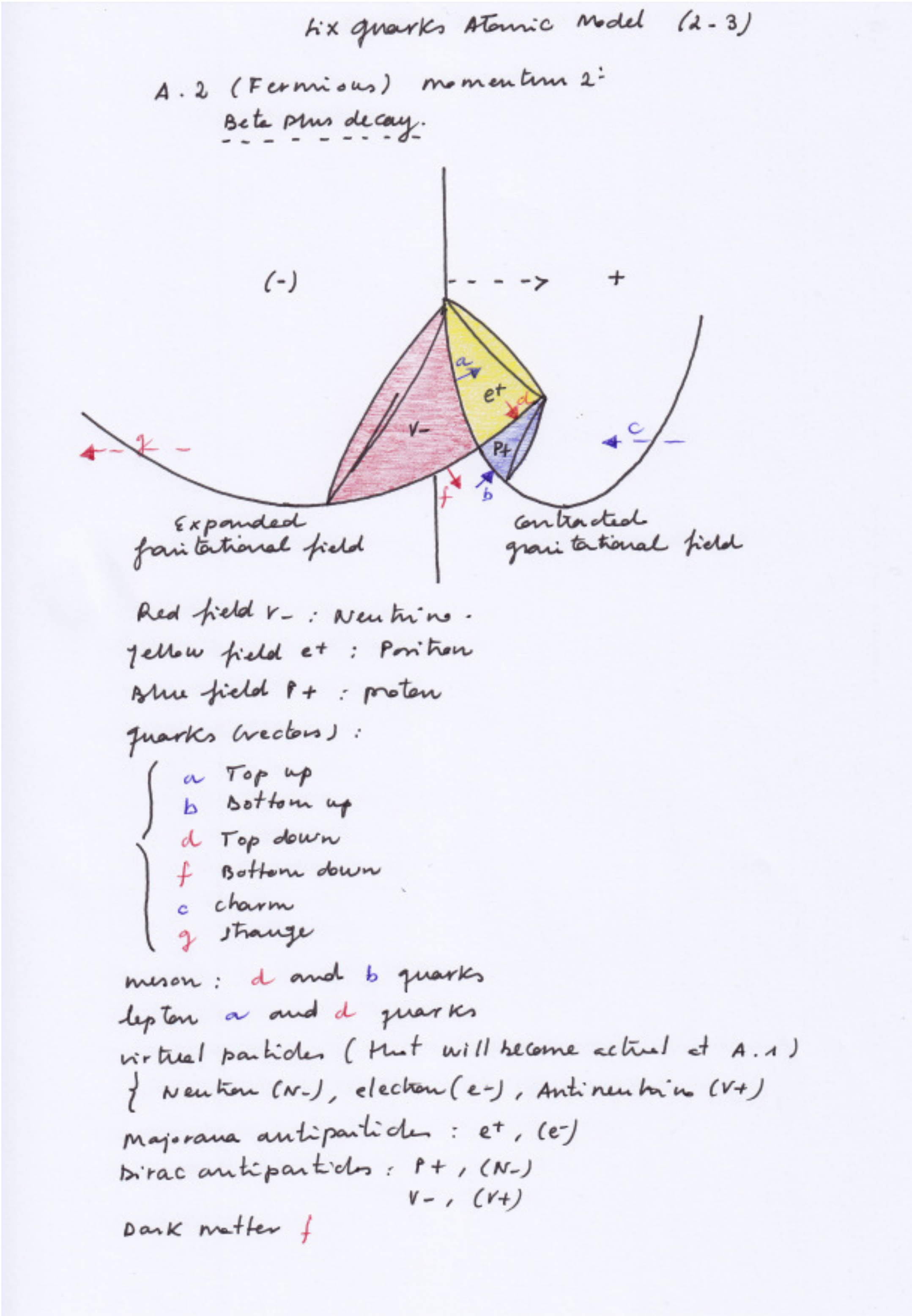
In this model, Gravity would be a force of pressure – idea already considered by Fatio and Le Sage, and even by Newton himself who was very aware that to speak about a force of attraction was not a mechanical explanation (see *"Pushing Gravity: New perspectives on Le Sage's theory of gravitation" – 2002, Matthew R. R. Edwards*) which would create the gravitational curvature. Expanding and contracting gravitational fields would be longitudinal waves.

The same could we said about the structure of an atom represented on the pictures bwlow:



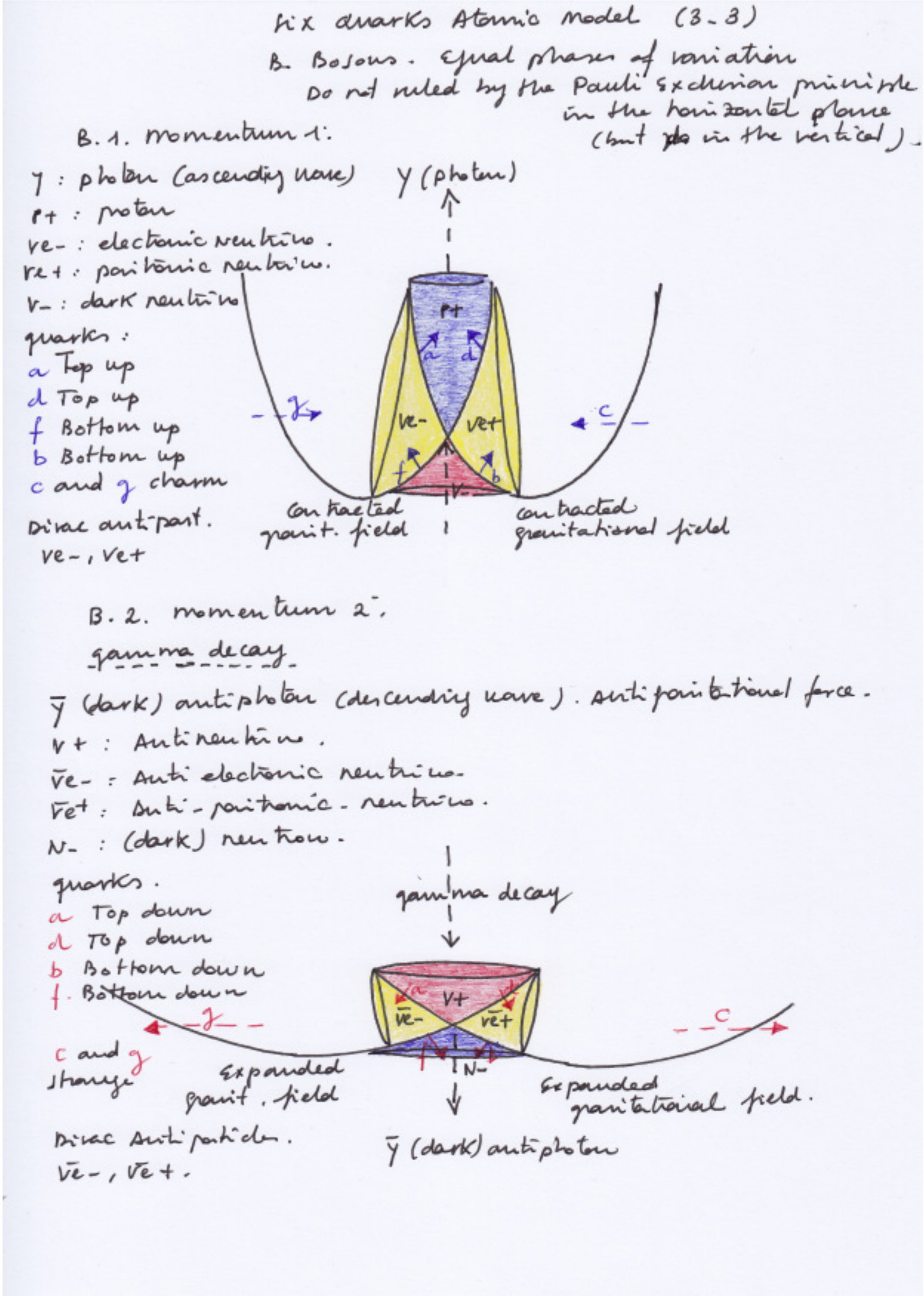
I think the atomic model could be represented and clarified by considering two intersecting fields that vary periodically, expanding and contracting. The fields created in and by their intersection would be the subatomic particles of the central and shared atomic nucleus.

When the intersected fields vary with opposite phase the electron field will be the field moving toward the left side where the gravitational fields contracts; its Majorana antiparticle – the positron – will be the same field moving toward the right side where the right gravitational filed now contracts (while the left one expands).



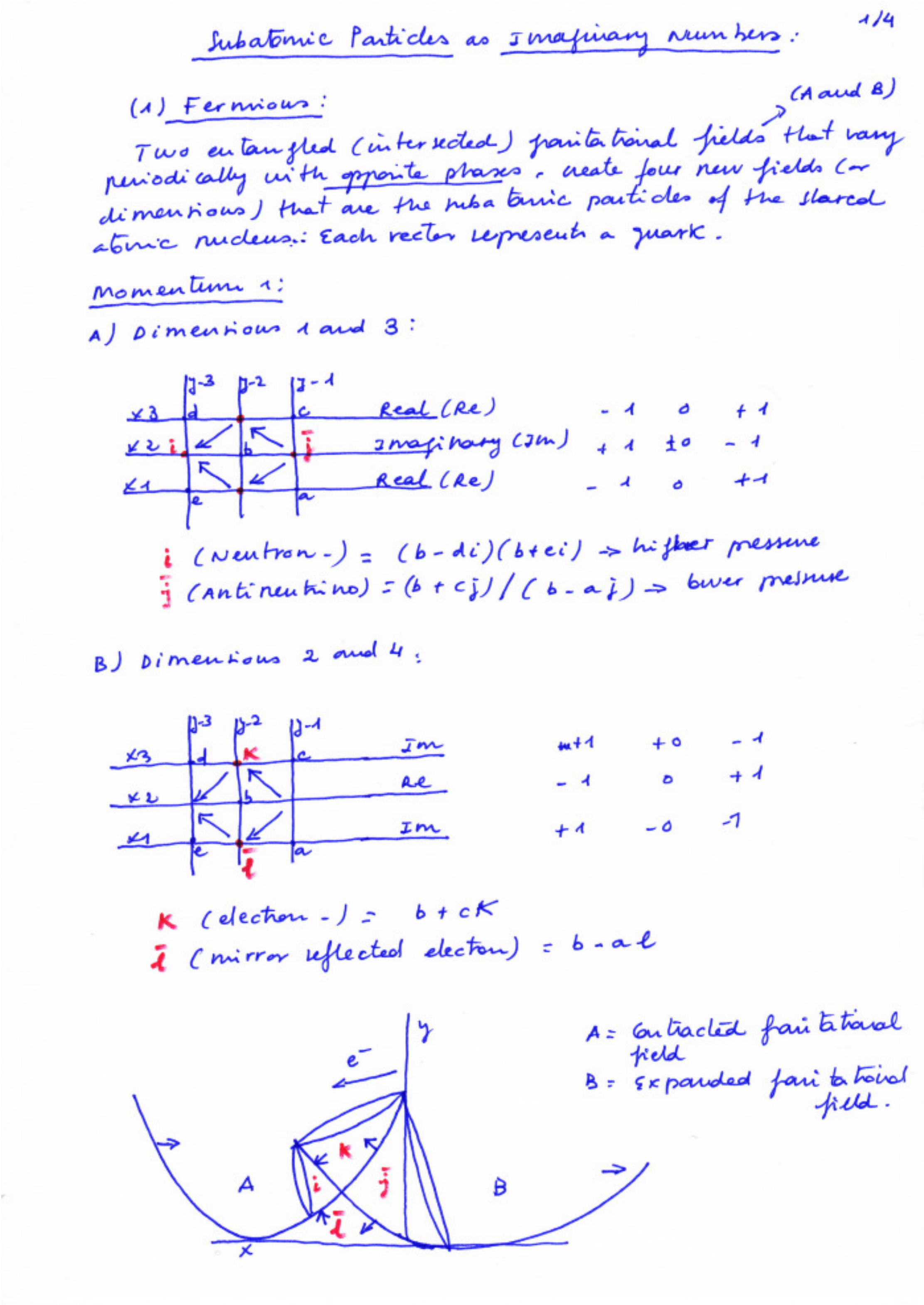
That explains that the positron is a "virtual particle" when the mention field acts as electron, and that the electron field is a "virtual particle" when that field acts as positron.

In these pictures you can see those different fields, how the neutron is the antiparticle of the proton, and how the mirror symmetry takes place in different successive times, and also how the mirror antisymmetry appears between the Neutron- antineutrino and Proton -Neutrino.

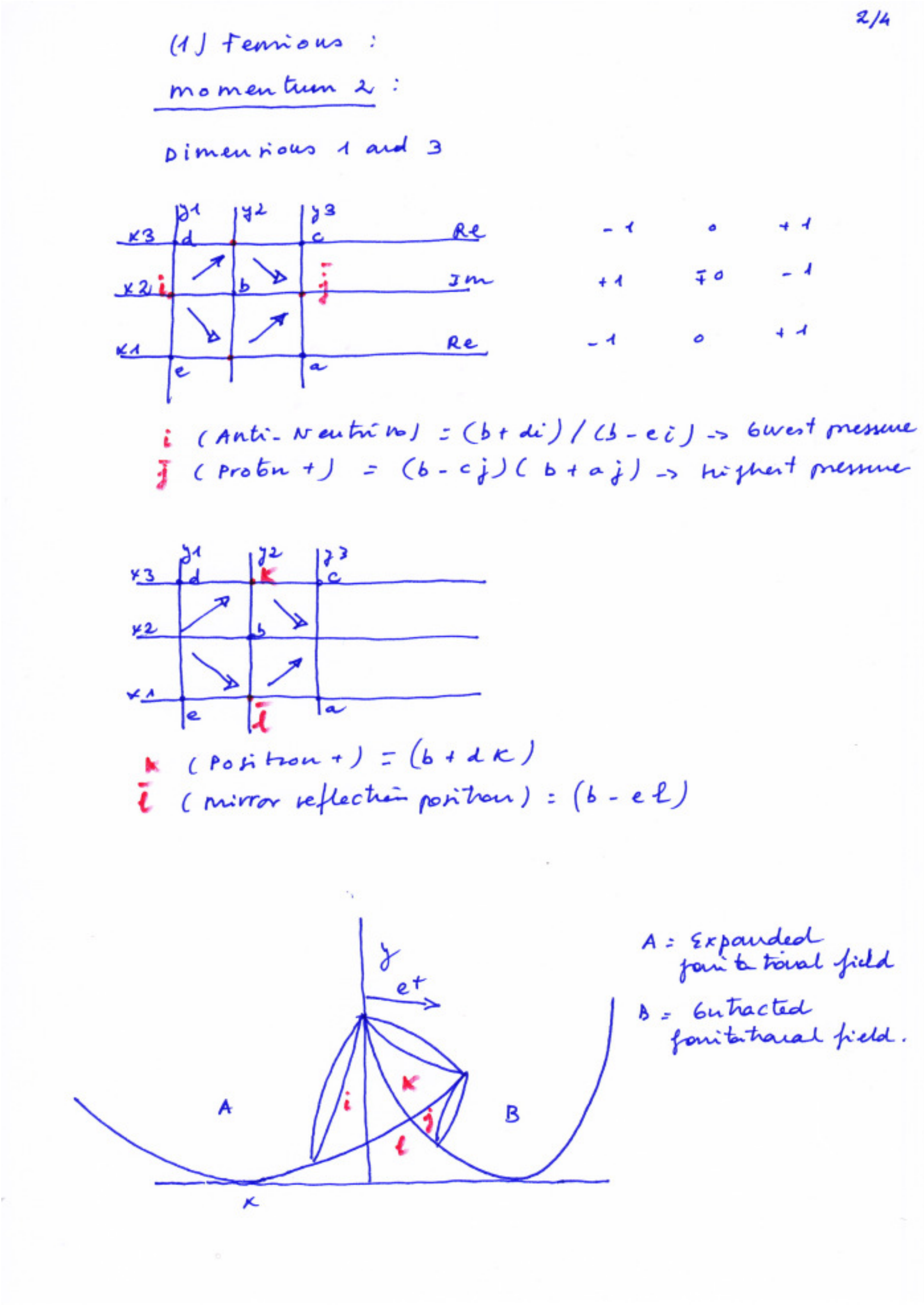


When the intersected fields vary with the same phase, when they both expands the ascending motion creates an ascending pushing wave which is a photon which decay when both intersected fields expands at the same time.

Here the force of pressure takes place up to down and follows a direction which is opposite by respect to the direction of the gravitational flux. When the decay of energy takes place, the anti gravitational force appears at the convex side of the intersected gravitational fields.



In this unconventional model subatomic particles appear as complex masses formed by the convergence on the X or Y coordinates of the called “quarks”, forces of pressure coming from the Y or X coordinates and vice versa. It is the same idea expressed above when it comes to understanding the complex zeros.



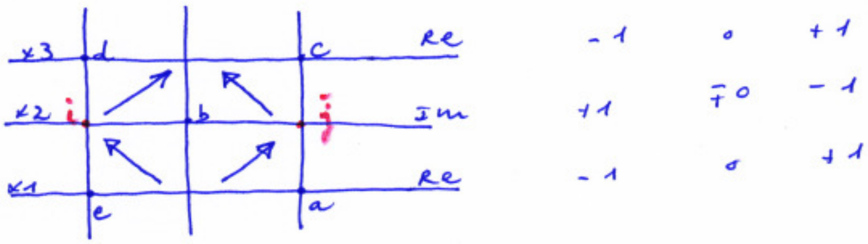
Subatomic Particles as Imaginary Numbers

(2) Bosons:

two entangled gravitational fields (A and B) that vary periodically with the same phase create four new fields (dimensions) that are the "subatomic particles".

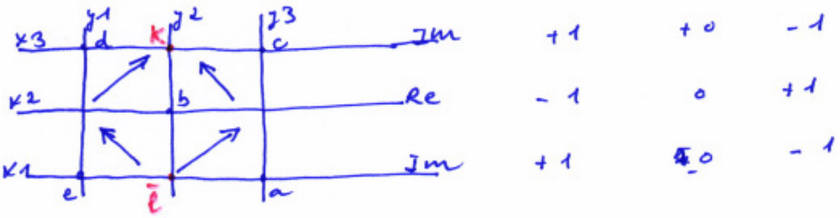
momentum 1:

Dimensions 1 and 3:

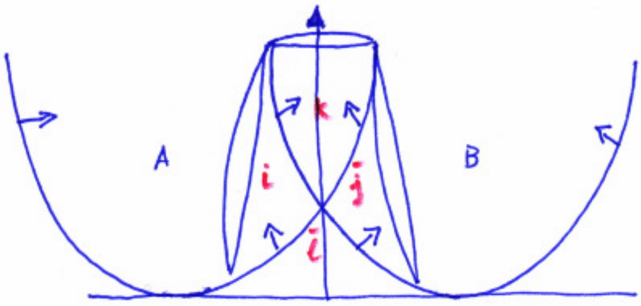


i (electron or "left handed electronic neutrino") = $(b + ei)$
j (Positron or "right handed electronic neutrino") = $(b + aj)$

Dimensions 2 and 4



K (proton -> photon) = $(b + dK)(b + cK)$
$\bar{l}$ (dark neutrino) = $(b + el)/(b + al)$



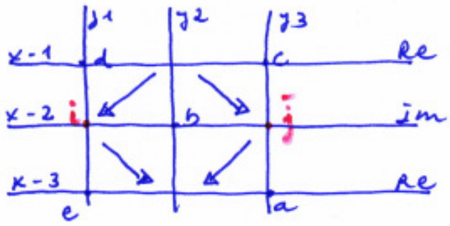
A = Contracted gravitational field

B = Contracted gravitational field

(2)- Bosons :

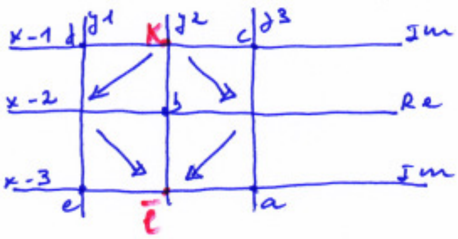
momentum 2 :

Dimentions 1 and 3 :



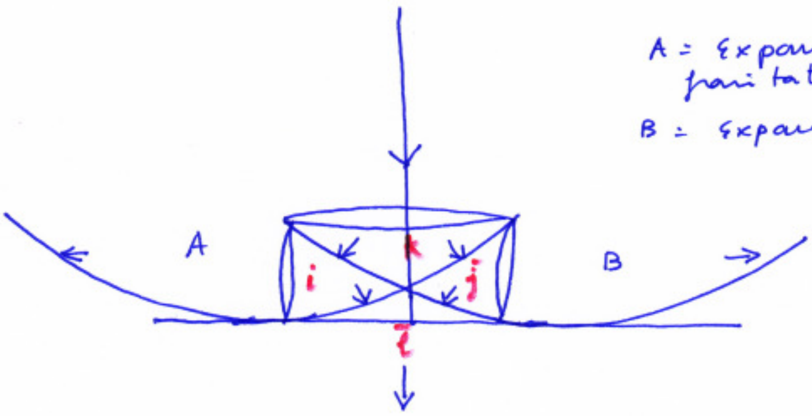
- 1	0	+ 1
+ 1	∓ 0	- 1
- 1	0	+ 1

i (Inverted electron) = (b - di)
j̄ (Inverted positron) = (b - cj)



+ 1	+ 0	- 1
- 1	0	+ 1
+ 1	- 0	- 1

K (Energy's decay : Anti Neutrino) = (b - dK) / (b - cK)
l̄ (Anti- photon = Antigravity) = (b - el)(b - al)
inverted proton↑



A = Expanded gravitational field
B = Expanded gravitational field

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I think the History of the whole mathematics of the last five centuries – and the physics they hold – has been totally conditioned and limited because of the lack of comprehension of what irrationality is and the aim of measuring and analyzing all things in Nature from a static and unique referential square.

If you read about the History of mathematics you will see that most times true innovative ideas were at the beginning ignored or considered heretical by the prevalent orthodoxy of that particular moment, or they remained totally misunderstood. Some times due to the lack of clarity of their developers, or because of their high abstraction, or because of the change of thinking they implied, or because of they challenged intimately assumed world-views of many people, etc. To change the deeply assumed ideas and believes, the paradigms deeply established at each time – and maths and physics are full of aprioristic believes – needs time. It also requires free people willing to make the effort of thinking by themselves.

More than thinking rationally and critically by themselves, human beings love to create myths and to admire the myths others created before, and to fetishize to their creators as if they were not persons but a kind of titans gifted with super human intelligences and supernatural powers. Maybe it is why so many times through the History it has been necessary to wait until the death of the "innovative" and "revolutionary" thinkers to see how their right or even wrong – always limited in any case – ideas were considered and fructified in many – also limited – ways. A death person is not already human.

None of the mathematicians – note that almost exclusively they have been men, which indicates the necessary limited and one-dimensional way in which maths has been developed because women have not participated on them – that thought the ideas that you where taught and you have internally assumed was not another thing than a simple and very limited person like you and me. Myths do not exist. Genius either. There are no intellectual giants. There are only people thinking by themselves and people who do not think by themselves.

Do not forgive, my dear, that you will always be able to think and to discover Nature by yourself.

Seek true instead of utility.

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Some very interesting bibliography:

- *"Patterns of Change". Linguistic Innovations in the Development of Classical Mathematics. Ladislav Kvasz.*
- *The Philosophical status of diagrams. Mark Greaves (which explains the reasons of the exclusion of diagrams and of the diagrammatic reasoning from the foundations of arithmetic).*
- *"The Mathematics of Plato's Academy" by David Fowler. It is better the second edition which has some clarifications and additions with respect to the fits one.*
- *The article "An invitation to read Book X of Euclid's Elements", by David Fowler.*
- *The article "The Croix des mathématiciens: The Euclidean theory of irrational numbers", by Wilbur Knorr.*
- *"Coloured Quadrangles: A Guide to the Tenth Book of Euclid's Elements" by Christian Marinus Taisbak.*
- *"Frege. A critical Introduction" by Harold W. Noonan. – "The Foundations of Arithmetic" by Gottlob Frege.*
- *"Theaethus", Plato's dialog where the idea of incommensurable magnitudes appeared fir the first known time. – Euclid. Books V and X. Translated by Sir Thomas L. Heath.*
- *"Le problème mathématique de l'espace. Une quête de l'intelligible", by Luciano Boi.*
- *Bernard Riemann-. On the Hypothesis Which Lie at the Bases of Geometry". Jürgen Jost Editor.*
- *Labyrinth of Thought. a History of Set Theory and its role in Modern Mathematics" by José Ferreirós.*
- *"The Search for Certainty. A Journey Through the History of Mathematics from 1800 – 2000" Edited By Frank Swetz. (Several authors).*
- *"Philosophy of mathematics and deductive structure in Euclid's Elements" Mueller.*
- *The article "Weierstrass' Construction of the Irrational Numbers"*
- *"Not Even Wrong" by Peter Woit. Vintage 2007. It explains clearly the development of theoretical physics from a mathematical point of view but without using any formula, and the problematic and stuck situation of current theoretical physics.*
- *"Pushing Gravity: New perspectives on Le Sage's theory of gravitation" – 2002, by Matthew R. R. Edwards.*