N=1 Supersymmetric Dual Quantum Field Model

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ABSTRACT: This paper introduces a supersymmetric dual-matter atomic model based on two intersecting fields that periodically vary in either the same or opposite phases, forming a shared nucleus of two transversal and two vertical subfields that represent the particles and antiparticles of the dual atomic nucleus. The bosonic or fermionic characteristics of the nuclear subfields are determined by their topological transformations, which are caused by the pushing forces generated by the negative or positive curvatures of the intersecting fields during their contraction or expansion, and by the periodical synchronization and desynchronization of the phases of the intersecting fields while rotating.

With a mainly visual and conceptual approach, the model employs a set of 2x2 complex rotational matrices of eigenvectors related in a modular way to Sobolev interpolations and to Tomita-Takesaki theory, graphically illustrating Reflection positivity, the Mass gap problem, the Jacobian conjecture, or the arising of a purely imaginary time dimension, between other topics.

The article first explains the fields model in a general way, then it introduces some mathematical formalisms, translates the general system to the standard atomic terminology, and finally compares the model with already known developments and theories.

KEYWORDS: Mass gap; reflection positivity; de Sitter vacuum; zero-point energy; Sobolev interpolation; supersymmetric quarks; Dirac spinors; gauge transformations; 2x2 rotational matrix; partial differentiation; complex conjugate functions; mirror reflection; symmetry; antisymmetry; Pauly exclusion; superposition; entanglement; two-time dimensions; Calabi-Yau manifolds; elliptic fibrations; string theories; T-duality; dark energy; Lobachevsky; Fourier; convolution; involution; Wick rotation; Minkowski; Tomita-Takesaki; modular matrices; Cartan-Killing pairing; Bethe matrices; Higgs bundle; exotic matter; positronium; protonium; Weyl semimetals; hidden sector; dark matter; Hitchin theory; Hyperkahler surfaces; Kummer quartic; Redox; acid-base reactions; Riemann-Silberstein vector; Jacobian conjecture.

1. Introduction

This study introduces an atomic model characterized by a dual nucleus comprising both matter and antimatter. The atomic system emerges from two intersecting fields that fluctuate with the same or opposite phases, synchronizing and desynchronizing periodically. The interactions between these fields give rise to a nuclear manifold consisting of two vertical subfields – one in the concave side of the intersection and another in its convex side – and two half-handed transversal subfields – left and right. The shape, mass, charge, inner kinetic energy, and spatial displacements of the four subfields will be determined by the pushing forces that the negative or positive curvatures of the intersecting fields generate while contracting or expanding.

Initially, the model will be introduced in a general context, subsequently translating it into the framework and terminology of quantum mechanics:

1.1 Antisymmetric system

When the intersecting fields have opposite phases, the left and right-handed transversal subfields will exhibit mirror antisymmetry: when the left transversal subfield expands the right subfield contracts, and vice versa. The two subfields are not interchangeable under rotation, they are noncommutative at the same time.

The top vertical subfield will move right to left, getting a negative sign, or left to right, getting a positive sign, toward the side of the intersecting field that contracts; moving in a pendular way left or right, this vertical subfield will be its own anti-subfield at different times.

The mirror transversal subfields are characterized by an antiphase relationship with each other, yet each of them maintains phase coherence with the intersecting space that encompasses it.

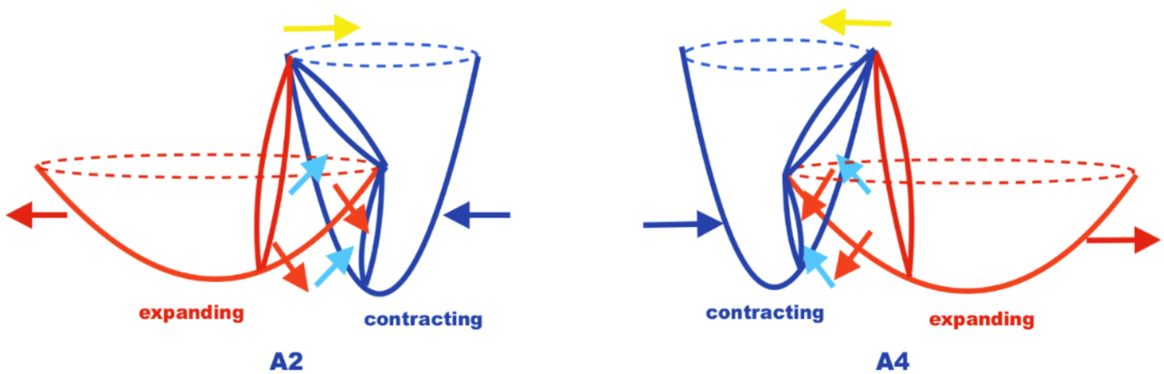

Fig. 1. Antisymmetric system fluctuating with opposite phases.

Fig. 1 shows the antisymmetric system in two different moments. In a first moment, represented by A2, the left and right transversal subfields are mirror antisymmetric – when the right subfield contracts the left one expands – and the top vertical subfield moves toward right. A moment later, represented by A4, the left and right transversal subfields remain antisymmetric, having commuted their expanding or contracting states, and the top vertical subfield moves toward left.

The left transversal subfield at moment A2 is mirror symmetric of the right transversal subfield at moment A4. And the top vertical subfield moving right at A2 is mirror symmetric of the top vertical subfield moving left at A4. When the top vertical subfield moves towards right at A2, it can be considered that it exists in a virtual way at A2, in the sense it has the potential to effectively exist a moment later, at A4, when moving toward the left.

Owing to their mirror antisymmetry, the states of the left and right transversal subfields at moment A2 (or at moment A4) are mutually exclusive: when the left transversal subfield contracts, thereby increasing its density and inner orbital kinetic energy, the right-handed transversal subfield expands, leading to a decrease in its density and inner kinetic energy.

In this sense, their states can be said to be governed by an “exclusion” principle.

The opposite states of the left and right transversal subfields are not superposed because, being mirror reflective subfields, they are different subspaces that reflect each other with a delayed or advanced phase of time.

Within the framework of a dual composite system such as the one proposed in this model, both “superposition” and “exclusion” must be interpreted in terms of mirror symmetry or anti-symmetry.

1.2 Symmetric system

In contrast, when the phases of vibration of the intersecting fields are equal, the transversal subfields will exhibit mirror symmetry simultaneously, being interchangeable under rotation. However, although they share the same phase, they will exhibit a phase opposition relative to the phase of the intersecting fields.

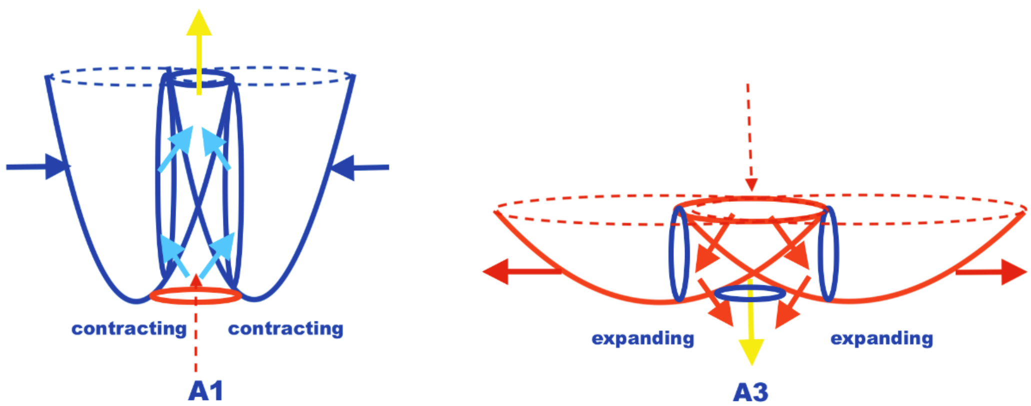

Fig. 2. Symmetric system fluctuating with the same phases.

Once the system exhibits mirror symmetry, the top vertical subfield aligns with the phase of the intersecting fields: when both intersecting fields contract, the top vertical subfield also contracts, ascending upward while emitting a pulsating force.

Subsequently, when both intersecting fields expand, the previously ascending subfield will decay while expanding. When such a decay occurs, an inverted pushing force is generated on the convex side of the system by the positive (or convex) curvatures of the expanding intersecting fields.

For a detector placed in the concave side of the system, the mass and energy that occur in the convex side of the intersection of the curved fields will be "dark" as directly undetectable.

1.3 Vectors of force and types of interactions

The pushing forces created by the contracting or expanding fields, with their inner negative curvatures or their outer positive curvatures, can be represented in a two-dimensional frame by means of 4 vectors.

Two converging vectors compressing a subfield imply a stronger force experienced by the contracting subfield whose volume decreases while its density increases, and its inner orbital motion experiences an acceleration or boost. The increased inner kinetic energy represents a greater bond that intertwines the intersecting fields in a stronger way.

Two vectors decompressing a subfield represent a weaker force experienced by the subfield, which expands in volume, decreases in density, and decelerates in inner kinetic energy. The decreased inner kinetic energy implies a weaker bond between the two intersecting fields. In that sense, it can also be said that the strong and weak interactions between the two intersecting fields allows the subfields of their shared nucleus remain united with a greater or weaker bond.

Alongside these strong and weak interactions, there will be an electric pushing force represented caused by the displacement of the vertical subspace when moving left or right in the antisymmetric system, or upwards or downwards in the symmetric system, and a magnetic force represented by their inner orbital motions.

The symmetric sand antisymmetric systems may also be interpreted as the electric and the magnetic moments, respectively, of a rotationally evolving system.

Additionally, the curvatures of the intersecting fields may be considered gravitational. They determine the mass and inner energy of their related subspaces.

2. MATHEMATICAL FORMALISMS

2.1 Rotational 2x2 complex matrices

The symmetric and antisymmetric manifolds can be considered either as two separate and independent systems, as two systems related by supersymmetric partners, or as two topological systems that are periodically transformed into each other by the periodical synchronization and desynchronization of the phases of vibration of the intersecting fields, forming a supersymmetric manifold. Here, only the latter case will be considered.

The complexity of system increases when considering it as a rotational structure. Let's examine the rotation of the system around its axis by means of a group of complex $2\text{x}2$ rotational matrices with a 90-degree rotation operator.

The four elements of the matrices are visually represented as eigenvectors, with eigenvalue 1 or -1, which reverse their directions when rotating the complex plane.

The four vectors actually change their position each time the whole plane rotates 90-degrees, but we can only distinguish that they have changed their direction. Such a change occurs when they commute their sign, which implies a multiplication of the eigenvector magnitude by 1 or -1, being flipped or reflected across its origin, or being permuted 180 degrees.

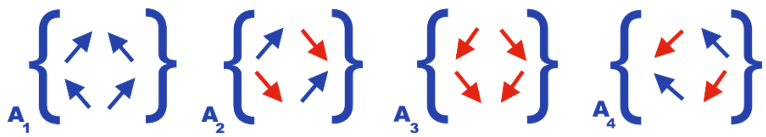

Fig. 3. Set of transformation matrices of eigenvectors

Fig. 3 shows the collection of the four transformation matrices that result when rotating 90º the complex plane four times, performing the operations of transposition, inversion and complex conjugation.

The identity matrix A1 represents the position of the eigenvectors of the symmetric system when the two intersecting fields, in phase, simultaneously contract.

- Rotating A1 by 90 degrees gives us the transpose matrix A2, whose eigenvectors represent the forces of pressure in the antisymmetric system when the left intersecting field expands and the right one contracts. A2 is also the partial conjugation of A1.

- Rotating A2 by 90 degrees gives us the negative reflection of A1, or A3. The A3 eigenvectors represent the forces of pressure in the symmetric system when the two intersecting fields expand with the same phase.
- Rotating A3 by 90 degrees gives us the transpose of A3, or A4. The A4 eigenvectors represent the forces of pressure in the antisymmetric system when the left intersecting field contracts and the right field expands. A4 is also the negative reflection of A2 and the second partial conjugation of A1.
- Completing a 360-degree rotation by rotating A4 by 90 degrees gives us A1, which represents the initial situation when both intersecting fields simultaneously contract.

2.2 Partial differentiation

The eigenvectors determine the curvature of the subfields, and their sign commutation is related to the topological transformation of those subfields. That transformation can be described with a complex or a conjugate function of four variables. In that sense the commutation of the eigenvectors is related to the evolution of that function.

This can be visually represented by considering that the slope of the eigenvectors being tangent to a point of a symbolic unit circle of radius 1. The commuted slope of the tangent eigenvectors represents a derivative related to the complex function represented by A1 and A3 matrices or related to the conjugate function represented by A2 and A4 matrices.

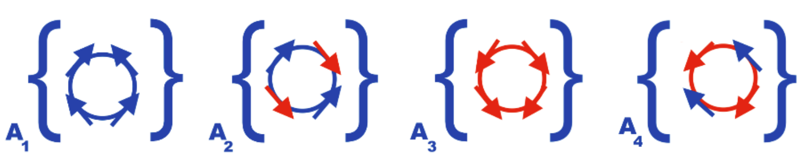

Fig. 4. Eigenvectors as tangent slopes of a unit circle in the rotational matrices

Each 90º rotation changes the sign of only two eigenvectors with respect to the previous matrix. In that sense, the transposition performed by A2 represents a ½ order derivative of A1, as only ½ of the eigenvectors – the ones related to its main diagonal – have commuted their sign.

A2 is a partial ½ complex conjugation of the complex matrix A1.

If the commuted eigenvectors represent the spin of the subfields in the antisymmetric system related to matrix A2, those subfields will have a noninteger spin with respect to A1. In this case, their mirror counterparts will be governed by an exclusion principle.

A3, the negative reflection of A1, represents an integer number of derivatives with respect to A1, as the four eigenvectors have already commuted their sign, although it only encodes a partial derivative with respect to A2 related to its second diagonal (upper left and bottom right eigenvectors).

Therefore, to obtain the complete first order derivation of A1 given by ½ + ½ order derivatives, it is necessary to rotate the matrix 90 degrees twice, having previously performed a partial conjugation of A1.

If the commuted eigenvectors represent the spin of the subfields in the symmetric system related to matrix A3, those subfields will have an integer spin with respect to A1. In this case, the transversal mirror symmetric subfields are not governed by an exclusion principle.

However, the top vertical subfield with integer spin and its reflection counterpart located at the convex side of the system will still be ruled by an exclusion principle because when the top subfield contracts at the concave side of the manifold, its mirror reflection subfield will expand at the convex side.

A4 encodes two positive eigenvectors with eigenvalue +1 with respect to A3. They represent a ½ order antiderivative with respect to A3. A4 also represents the whole first order derivative of A2 (given by ½ + ½ order derivatives); A2 and A4 together form a whole complex conjugation with respect to A1. Therefore, to obtain the complete complex conjugation of A1, it is necessary to rotate the matrix 90 degrees three times.

A1 represents the first order antiderivative of A3 (given by the ½ + ½ order antiderivatives), and the ½ antiderivative of A4. Therefore, to revert A1, the matrix must be rotated 90 degrees four times.

The rotational dynamic of the eigenvectors represented in this group of complex matrices, seems to imply that the smooth evolution of the symmetric system represented by A1 (when the intersecting fields contract) and A3 (when a moment later the intersecting fields expand), loses its linear continuity by being interpolated between A2 and A4.

In this sense, if the symmetric system is described by a complex ordinary differential equation and the antisymmetric system is described by the conjugate solution of the differential equation, then those separate equations can only describe the evolution of the physical states and displacements of half of the dual system. In that case, the system may be incompletely described and could be defined only by statistical methods.

2.3 Rotational interpolation

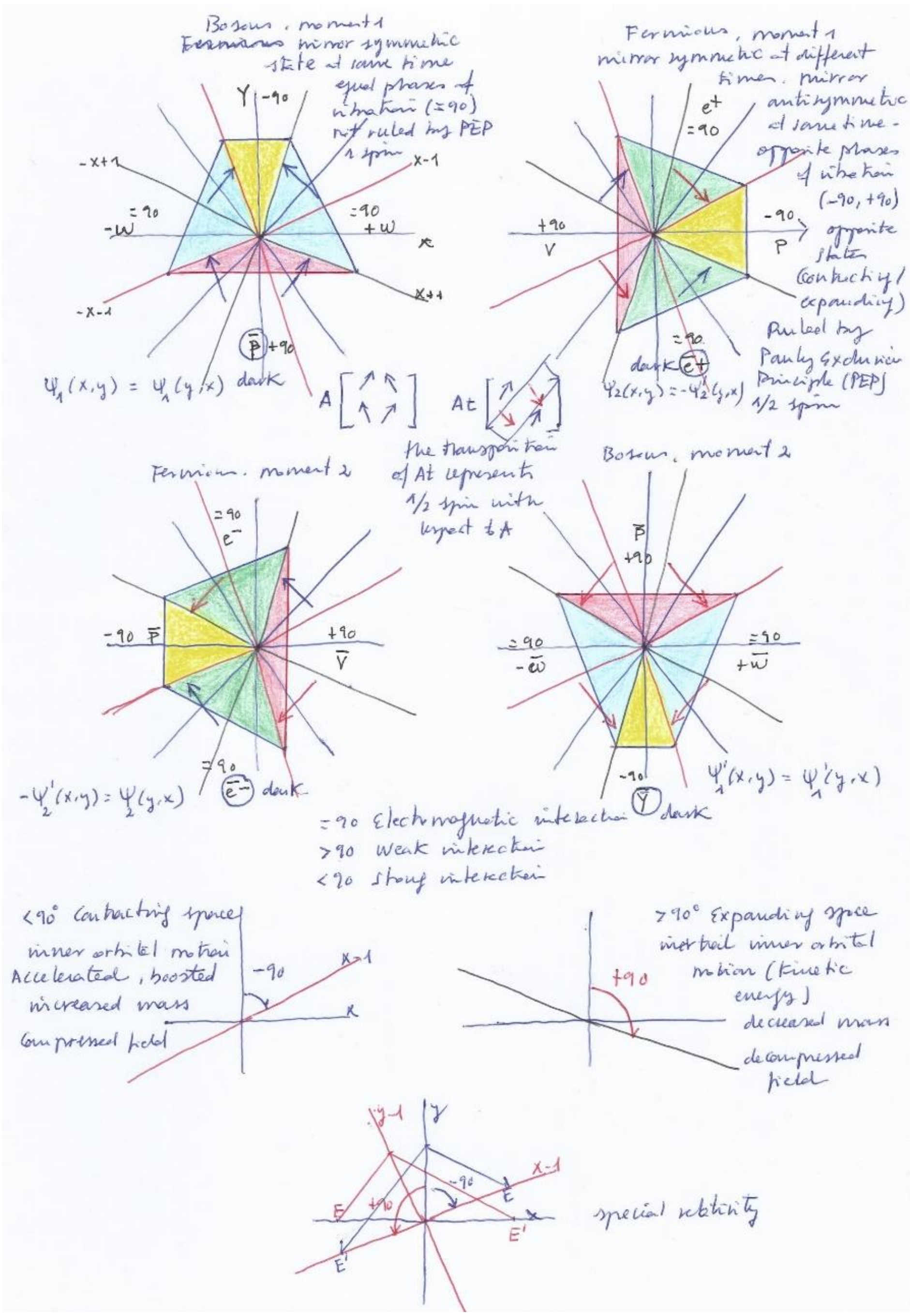

Fig. 5. Rotational system, visual representation

Diagram, Fig. 5, represents the fractional commutation of the eigenvectors embedded in the rotational nucleus of subfields shared by the intersecting fields.

The subfields depicted change their shape when the 90-degree rotation is performed, contracting or expanding, moving left or right, or ascending or descending – because of the fluctuation of the intersecting fields.

However, the conformal or nonconformal nature of the model is not clear enough, as it will be mentioned in the model inconsistencies.

The interpolation between the symmetric and antisymmetric systems may be related to Sobolev interpolations [1], where "spaces of functions that have a noninteger number of derivatives are interpolated from the spaces of functions with integer number of derivatives".

In the antisymmetric conjugate system, Sobolev embedding may be represented by the right contracting transversal subfield at moment t (matrix A2), being virtually embedded inside of the right expanding transversal subfield at moment t' (matrix A4):

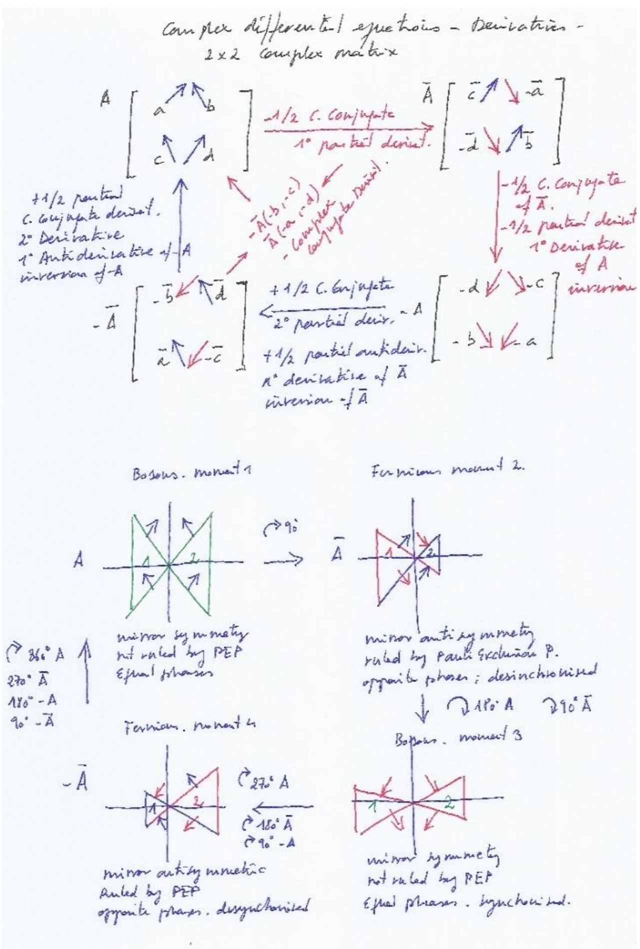

Fig. 6. Matrices interpolation

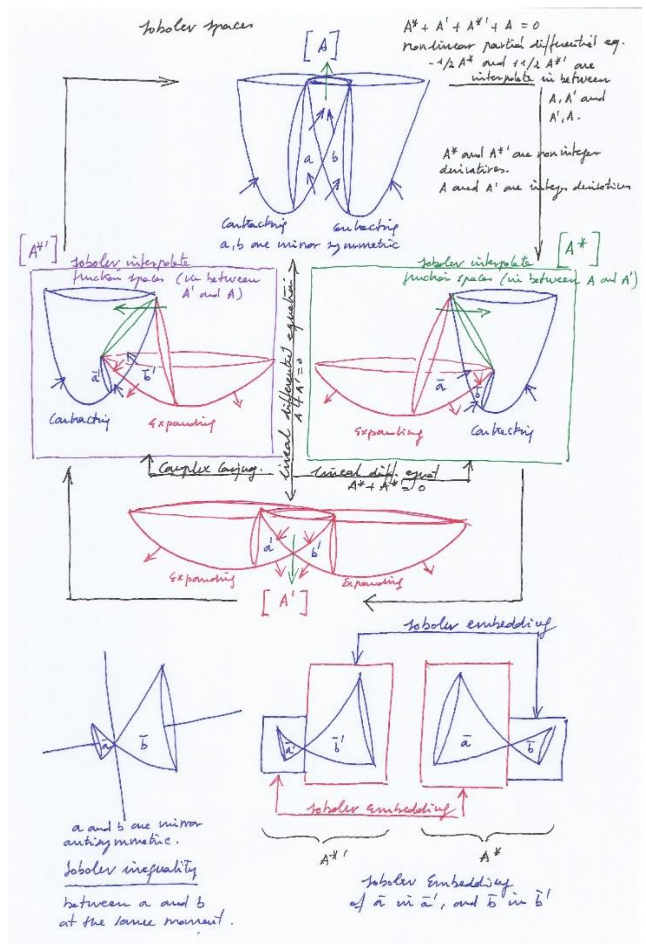

Fig. 7 Sobolev function spaces interpolation

2.4 Convolution

The combination of the complex ordinary function of four complex variables, represented by matrices A1 and A3, and the complex conjugate function represented by matrices A2 and A4, can also be described as a convolution [2].

By adding the products of the four transformation matrices that represent partial conjugations of the previous state, the identity matrix that represents the complex function is obtained.

2.5 Harmonic functions

The conjugate function given by A2 and A4 is a conjugate harmonic of the complex function, and vice versa. The complex and the conjugate function are, in the context of the rotational system, interdependent and cannot exist without each other.

Antisymmetry arises in the conjugate system of A2 and A4 by introducing a change in phase in one of the sides of the reflection, while the other side keeps following the unchanged phase of A1 and A3. This change in phase can occur by a gradual desynchronization, or suddenly, when a rotation occurs changing the sign of ½ eigenvectors.

The addition of the main and harmonic phases can be performed with a Fourier transform [3].

2.6 Bäcklund transformations

The constructive interdependence of the complex and conjugate functions represented by the complex A1 A3 and the conjugate A2 A4 matrices respectively can be interpreted as well as Bäcklund transformations, where the conjugate function transforms the complex function and vice versa.

The prototypical example of a Bäcklund transform [4] is the Cauchy-Reimann system, where "a Bäcklund transformation of a harmonic function is just a conjugate harmonic function".

2.7 Operator algebras

The symmetric and antisymmetric systems can also be described as two independent groups of cyclic eigenvectors that form two von Neumann algebras: an antisymmetric automorphic algebra and a symmetric automorphic algebra, which imply antisymmetric and symmetric mirror reflection algebras, respectively.

However, both independent algebras can be related by means of modular combinations, which, in the context of our rotational matrices and interpolated functions, are the combinations of the transform matrices whose operations imply fractional derivatives or antiderivatives.

Modular combinations of von Neumann algebras are studied by Tomita-Takesaki (TT) theory [5].

In TT theory two intersecting algebras form two shared "modular inclusions" (with + - "half sided" subalgebras) and a "modular intersection" (with an "integer sided" subalgebra).

The left and right half handed subalgebras will be images of each other, when they are commutative, or they will not be their mirror image when they are noncommutative. Mapping the modular inclusion to its reflection image, the left and right subalgebras will be the opposite image of each other (reverting their initial signs) if they are commutative; if they are noncommutative, the initial left sided subalgebra will be the image of the right sided mapped subalgebra, and the initial right-handed subalgebra will be the image of the left sided mapped subalgebra.

TT theory decomposes a linear transformation into its modular building blocks, revealing automorphisms.

Decomposing the bounded operator, it obtains the modular operator and the modular conjugation (or modular involution) which is a transformation that reverses the orientation, preserving distances and angles.

Translating the abstract algebraic terms to the fields model, two intersecting algebras would represent the two intersecting fields fluctuating with the same or opposite phase.

The half handed subalgebras (or "modular inclusions") will be the transversal subfields of the nucleus shared by the intersecting fields, while the integer handed subalgebra (or "intersection inclusion") will be our vertical subfields. In this context, we identify commutativity and noncommutativity with mirror symmetry and mirror antisymmetry, respectively.

The bounded operator that is decomposed will be the 90-degree rotational matrix; The modular building blocks are the set of matrices that are obtained when applying the operator. The modular operator will be the ½ partial conjugate A2 matrix; And the modular conjugation will be the conjugate matrix A4, which forms the whole conjugation by adding the fractional conjugations ½ + ½.

Therefore, by separating the conjugate matrix from the complex one the automorphism of the antisymmetric conjugate system is found.

The half sided algebras that form a modular inclusion are noncommutative, it means we are in the antisymmetric system where the left intersecting field contracts while the right one contracts and vice versa; in that system, the left transversal subfield will be the mirror symmetric image (it will be the mapped image) of the right transversal subfield when, later, the left intersecting field expands and the right one contracts.

In that sense, a past half handed subalgebra is being mapped with its future image. A time delay will exist between both subalgebras.

Considering Δ as the modular operator A2, J the modular conjugation A4, and M the intersection of two Von Newmann algebras, Δ^-Yt M Δ^it will represent the positive and negative ½ sided modular inclusions of the modular operator, being t a real time dimension and it an imaginary time dimension given by the partial conjugation of A1 or A3.

It is this different time dimension what makes noncommutative, as non-interchangeable, the modular + and - inclusions related to Δ in the antisymmetric system.

Applying the modular involution, yields J^yt M' J^-it.

Δ^-yt is transformed into J^yt and Δ^it is transformed into J^-it', being J^yt M' J^-it the involutive automorphism of Δ^-yt M Δ^it.

The noncommutative, as non-interchangeable, Δ^-yt and Δ^it become commutative or interchangeable through time at J^yt M' J^-it, fixing their antisymmetry in that way.

The same type of operations can be performed by taking A2 as the identity matrix. Rotating clockwise, A3 would be the modular operator and A1 the modular conjugate automorphism.

2.8 Vertex operators

Among the numerous mathematical developments relevant to the intersecting model, we can also highlight the vertex operators' formalism [6], where fields are inserted at specific locations of a two-dimensional space.

In the introduced fields model, the vertex point is the point of intersection between the left and right intersecting fields.

This intersection point would represent a unified coupling gauge point, which will be displaced upward or downward in the symmetric system or leftward or rightward in the antisymmetric system."

2.9 Reflection positivity

Related to the delay in time in the antisymmetric system, it can also be mentioned a property that all unitary quantum field theories are expected to hold: "reflection positivity" (RP). [7]

The positive increasing energy that appears in one side of the mirror system should also be reflected in the other side. However, in the context of the antisymmetric system, the positive or increasing energy of the contracting right transverse subfield does not mirror simultaneously in the expanding left transverse subfield, which exhibits negative or decreasing energy.

Therefore, to obtain a positive energy reflected at the left side, making the sides of the system virtually symmetric, a time reversal operation is needed.

To observe the positive energy reflected at the left side, it will be needed to go back in time to the moment where the left transversal subfield was contracting and had a positive energy. This operation is performed by a type of "Wick rotation". [8]

The main time phase of the symmetric system can be represented with the Y coordinate.

By performing a partial conjugation that involves a fractional derivative, the time coordinate Y undergoes a rotation into the purely imaginary dimension within the complex plane. At that moment, the mirror system becomes antisymmetric as one side of the system keeps following the imaginary time of Y while the other side follows a harmonic phase. A positive or negative time lag has been introduced.

Reversion time on one side of the system serves as a symbolic tool to virtually restore symmetry to the time phases. To revert to the previous time, one could perform a reverse rotation of the complex time axis (X +iY) to achieve a full complex conjugation at (−X −IY).

In the A matrices context, that time backwards rotation represents an antiderivative of -A.

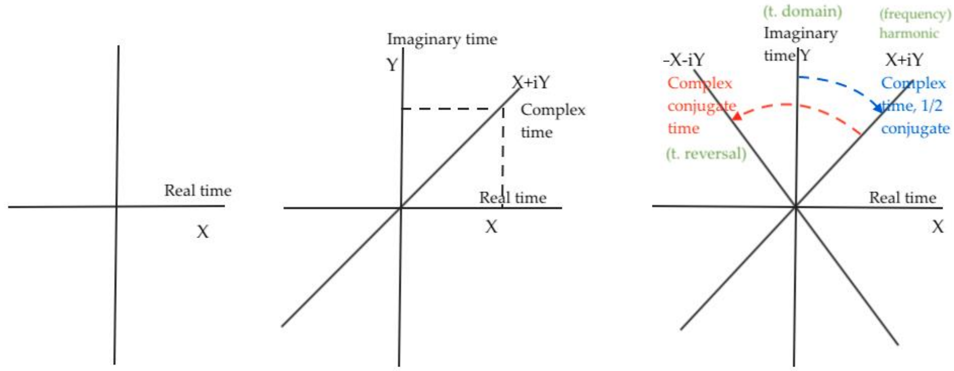

Fig. 8. Rotational time backwards and forwards

When the time reverse has been symbolically completed, in the left side of the mirror system the left subfield will be contracting, having an increased positive energy; this is a past reflection of the future positive energy that there will be a moment later in right side.

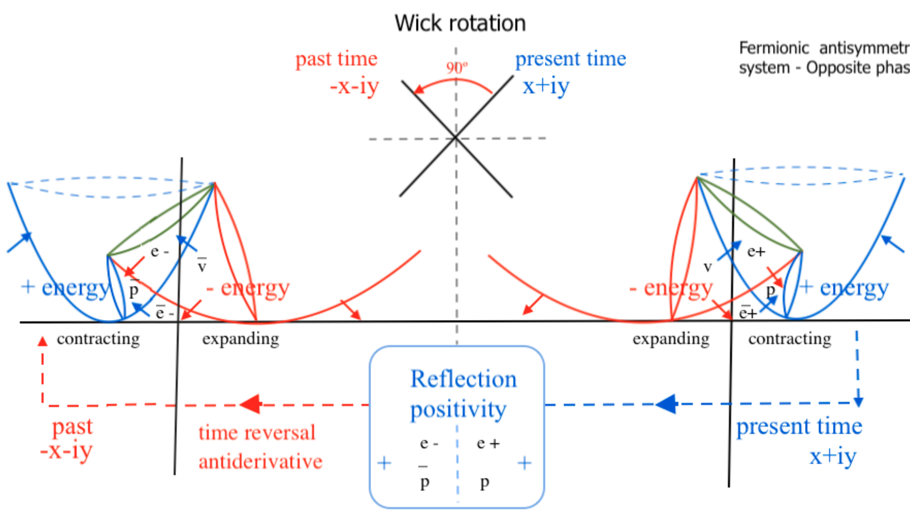

Fig. 9. Reflection positivity in the antisymmetric system

In the reverse past time, at the right side of the system the right subfield will be expanding having a decreased negative energy.

In regard to the symmetric system, positivity is reflected between the right and left transverse subfields at the same time. In that sense, it's not necessary to use the Wick operation to reverse time. Both left and right transversal subfields will be the mirror reflection of each other at the same time.

However, in the case of the strong interaction in the symmetric system, when the contracting vertical subfield has an increased positive energy while ascending to emit a pushing force, it will be necessary to virtually visit a past moment to look for a previous state where positivity could be reflected.

Going back in time, the vertical subfield will be losing its energy while expanding, moving downwards. Therefore, at that past moment, the vertical subfield will not display a positive energy.

Reflection positivity, however, can be found at that past moment in the convex side of the system of the two intersecting fields, where an inverted subfield with convex curvatures will be experiencing an increased energy.

That inverted subfield can mirror the vertical subfield which in a future state will be ascending in the concave side of the system through the Y axis.

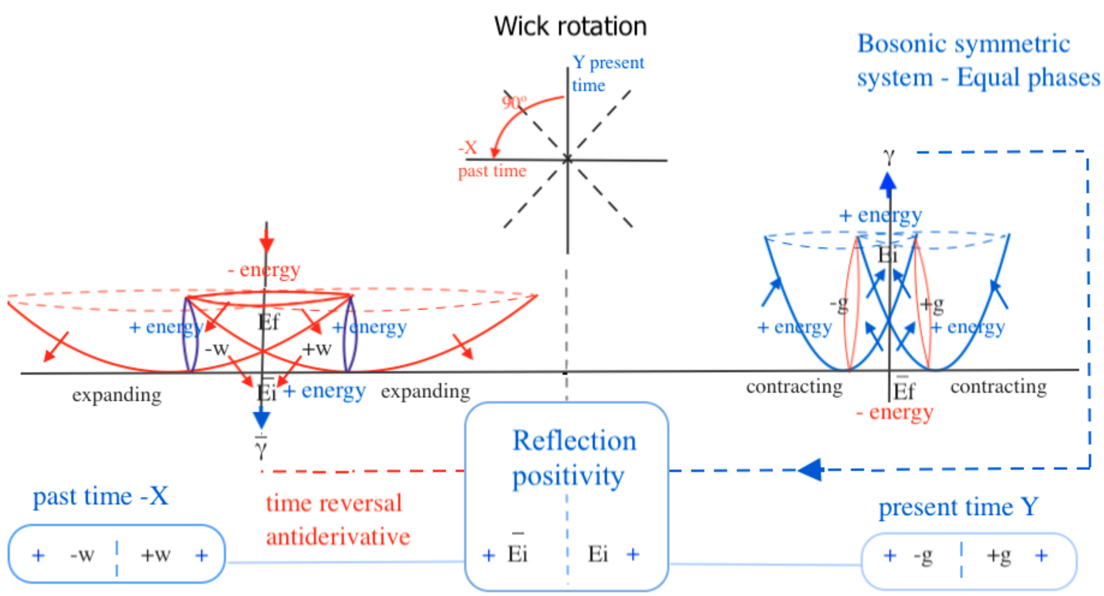

Fig. 10. Reflection positivity in the symmetric system

The missing reflection positivity in the concave side of the system in the strong interaction can be related to a mass gap problem when it comes to the weak interaction.

2.10 Mass gap problem

There will be a mass gap [9] in the system when the two intersecting fields simultaneously expand, and the vertical subfield experiences a decay of energy.

This case represents the ground state with the lowest possible energy of the vertical subfield, which is always greater than 0 because the highest rate of expansion of the intersecting fields prevents them from having zero curvature.

The zero point of the vacuum, where there should be no energy nor mass, is placed at the point of intersection of the XY coordinates, and that point is never reached by the vertical subfield that descends through the Y axis while expanding during its decay.

An "upper" mass gap would be referred to the highest possible mass of a particle in the strong interaction. Its limit would be given by the greatest rate of contraction of the intersecting spaces.

Fig. 11 represents graphically the mass gap in the symmetric system; the upper gap occurs in the compressed photonic subfield when both intersecting fields contract, while the lower gap occurs in the decompressed subfield when both intersecting fields expand:

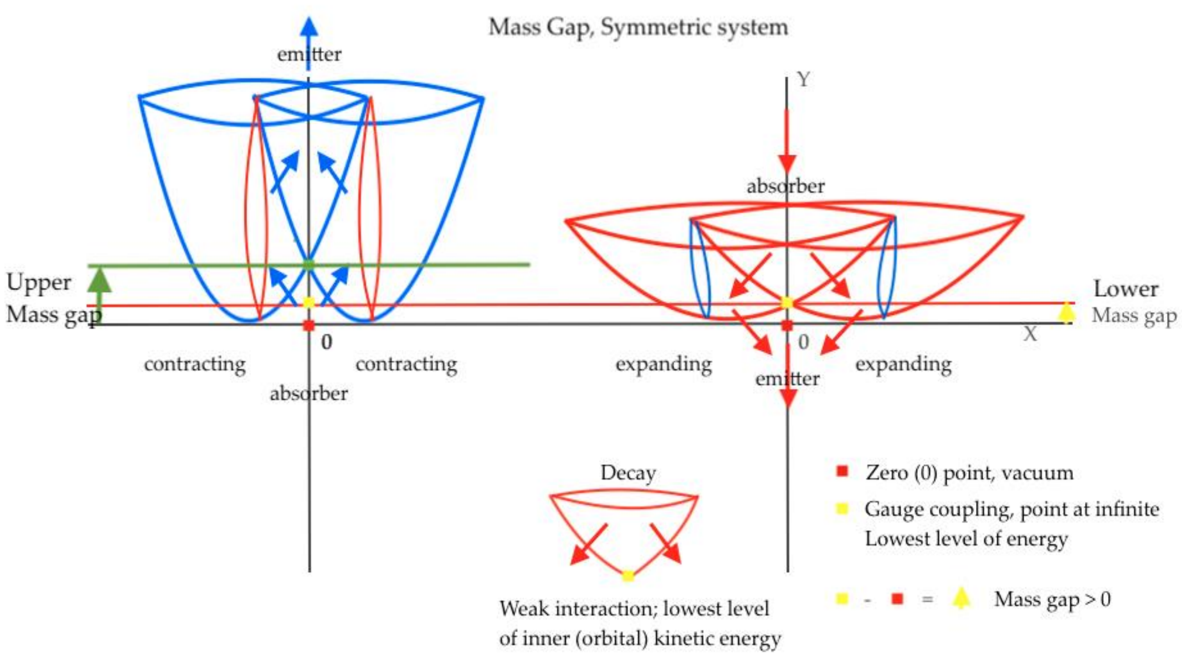

Fig. 11. Mass gap in the symmetric system

The zero point of the vertical subfield is marked in yellow on the above diagram, at the point of intersection of the left and right intersecting fields.

The gap is given by the distance from that point to the zero point where the X and Y coordinates intersect, represented by a red mark. An arrow shows the gap distance between those critical points.

However, in this model, the zero point does not represent a vacuum where neither energy nor mass exists.

When the mass and energy of the vertical subfield reach their weakest level in the concave side of the symmetric system, an equivalent amount of energy and mass arises in the convex side, where the zero point is located, as the result of the double pushing force caused by the displacement of the positive curvature of the expanding intersecting fields.

That mass and energy at this zero point will be considered dark from the point of view of the concave side of the system.

In the antisymmetric system, the lowest energy level occurs when a transverse subfield experiences a double decompression due to the displacement of the concave curvature of the contracting intersecting field and the displacement of the positive curvature of the expanding intersecting field.

The corresponding double compression is then experienced by its mirror antisymmetric transverse subfield.

Fig. 12. Represents visually the map gap in the antisymmetric system, with the left and right displacements of the point of intersection:

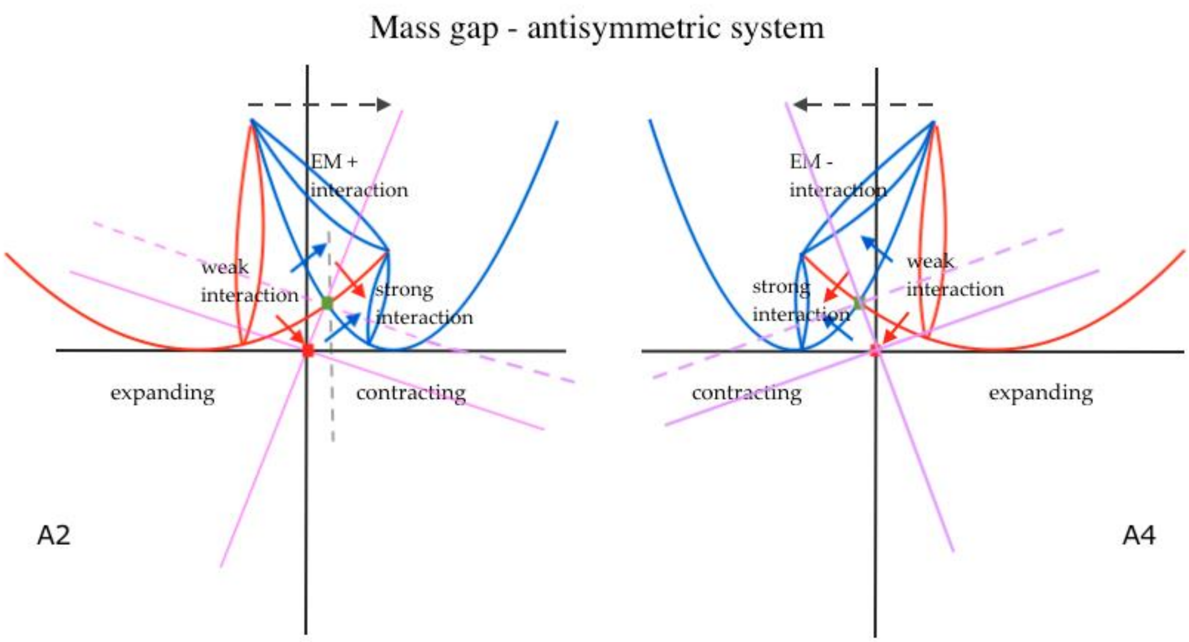

Fig. 12. Mass gap in the antisymmetric system

2.11 Jacobian conjecture for four variables

The Jacobian conjecture [10] states that if a polynomial function from an *n*-dimensional space to itself has Jacobian determinant which is a non-zero constant, then the function has a polynomial inverse.

A1 and A3 can be considered as Jacobian matrices that contain the first order derivative of the complex vector-value function of four variables. A2 and A4 can be considered as Jacobian matrices that contain the first order derivative of the complex conjugate vector-value function of four variables. The constant determinant would be the 90-degree rotational operator, which causes the commutation of the signs of the eigenvectors that represent the matrix elements with eigenvalue 1 or -1.

The eigenvectors are the four variables of these Jacobian functions. They represent the forces of pressure caused by the negative or positive curvatures of the intersecting fields while contracting or expanding, determining by pairs the displacements, curvatures, and physical properties of the subfields.

However, the rotation of the whole system implies an interpolation between the complex and the complex conjugate functions. In that way, to operate the first order derivative that causes the inversion and the negative reflection of the antisymmetric system, mapping A2 to A4, it will be necessary to operate a ½ order derivative, plus a ½ order antiderivative given by a partial conjugation of the positive antisymmetric system represented by A2, and a partial conjugation of the negative symmetric system represented by A3.

In the antisymmetric system, the A2 right contracting transverse subfield at moment T1 is mapped to the A4 left contracting transverse subfield at moment T2, and the left expanding subfield A2 at T1 is mapped to the right expanding transverse subfield A4 at T2. In the rotational framework, the applied mirror reflection implies that the subfields physically interchange their positions after a 180-degree rotation, resulting in an inversion of A2 at A4. The antisymmetric system A4 at T2, which maps the antisymmetric system A2 at T1, represents its negative reflection.

However, when the phases of vibration of the intersecting fields are equal, and so the transverse subfields exhibit mirror symmetry simultaneously, a 180-degree rotation will not map the left and right transverse subfields to each other because the transverse subfields will be contracting after the 180-degree rotation and are therefore not isometric to the unrotated subfields. However, in this case, the vertical subfield will have been mapped to an inverse vertical subfield located on the convex side of the system.

The mirror transverse subfields are described by the same spatial dimension, which cannot be the same dimensions related to the intersecting fields, because the Y coordinate of the transversal subfields will correspond to a diagonal axis in the intersecting fields. Using the same referential metric for the dual system and the subsystem will introduce an irrational elongation of the spacetime.

In the antisymmetric system, one of the transverse subfields will follow a delayed time that represents a purely imaginary time dimension, distinct from the non-delayed real time followed by the other-handed transverse subfield.

The smooth quantization of the system is introduced by the periodic 90-degree rotation, which creates the interpolation between the complex function describing the symmetric space system related to matrices A1 and A3, and the harmonic conjugate function describing the antisymmetric space system related to matrices A2 and A4.

Both vectorial functions have four variables. However, each 90-degree rotation entails a partial conjugation relative to the previous situation, as only two of the four eigenvectors will exchange their signs. This implies a ½ order derivative or a ½ order antiderivative. The map of the antisymmetric system requires the inversion provided by a ½ derivative (exchanging the signs of two eigenvectors with respect to the positive symmetric system) plus a ½ antiderivative (anti-exchanging the signs of two eigenvectors with respect to the negative reflection of the symmetric system).

- A1 (0-degree rotation) represents the eigenvectors in the symmetric system, when the transversal subspaces have mirror symmetry at the same moment; performing its partial conjugation (rotating the plane 90-degree) only two eigenvectors (acting as two variables) change their sign at A2.
- A2 (90-degree rotation) represents the eigenvectors when half of the system has a delayed its phase, introducing a purely imaginary time dimension) having mirror antisymmetry with respect to the other half side. A2 represents a ½ order derivative of A1.
- A3 (180-degree rotation) represents the partial conjugation of A2 (only the yet two uncommuted eigenvectors commute now their sign with respect to A2); A3 also represents the negative reflection of A1; its four eigenvectors (acting as four variables) have already commuted their sign with respect to A1; A3 represents the ½ order derivative of A2, and the first order (½ + ½) derivative of A1.
- A4 (270-degree rotation with respect to A1, 180-degree with respect to A2, and 90-degree with respect to A3) represents the transpose of A3, the ½ order antiderivative of A3, the second transposition of A1, and the first order (½ + ½) derivative of A2; A4 is also the negative mirror reflection of A2, having commuted their sign the four eigenvectors.
- An additional 90-degree rotation produces A1 which represents the positive reflection of A3, a ½ order antiderivative of A4, and the first order (½ + ½) antiderivative of A3.

Then, considering A2 as a starting point, the rotational inversion operated to obtain its negative map at A4 implies two partial conjugations, the partial derivative of A2 given by A3, and the partial antiderivative of A3 given by A4. In that sense, to obtain the map of A2 it's necessary to perform the partial inversion of A3.

The dual fields model then suggests a relation between the Jacobian conjecture for four variables and the partial commutation of the four eigenvectors of pairwise commuting matrices associated to the polynomial map related the complex function, and the polynomial map related to the interpolated harmonic complex conjugate function.

In that context, the inversion proposed by the Jacobian conjecture would be related to Sobolev interpolations, Fourier inverse, Reflection positivity, and Tomita Takesaki modular conjugation in the terms previously seen.

2.12 Representation theory

In the context of representation theory, we may also interpret the antisymmetric system of subspaces represented by the matrix A2 as an original vector space, and its negative mirror reflection matrix A4 as a dual vector space.

To connect A2 and A4, forming their isometric automorphism through time, it is necessary to pass through A1 (the identity matrix) and A3 by means of the fractional differentiation or partial conjugation given by the transformation matrices when the 90-degree rotational operator is applied.

The bridge that fixes the gap between A2 and A4 would be related to the Langlands parametrization.

The Langlands dual group that allows the creation of the automorphism between the subspaces of the antisymmetric system will be represented by the conjugate matrices A2(90º) and A4(270º), and the complex matrices A1(0) and A3(180).

The two eigenvectors that determine the curvature of each transversal subspace and the sign commutation of ½ of each pair, causing the transformations of the system, would be related to Cartan-Killing pairing.

Additionally, considering as an example the antisymmetric system where the left transversal subspace L contracts at t1 while the right transversal subspace R expands at t2, and the left transversal subspace expands at t1' while the right one contracts at t2', it can be suggested that the expanding Lt1' will be the functional Galois extension of Lt1, and Rt2 will be the functional Galois extension of Rt2', connecting the intersecting fields model as well with the Langlands program.

The collection of 2x2 complex matrices may also be related to Bethe [11] transfer matrices of eigenvectors related to complex and conjugate functions in the context of quantum integrable systems.

The unit circle mentioned at the beginning may be interpreted as a flattened version of the unit sphere, which is a visual way of representing geometrically the rotations of nuclear spins in ½ particles. This unit sphere is also related to the Bloch theory. [12]

2.13 The role of pictures in Klein's research

On the other hand, it can be interesting to mention that "a significant role in Klein's research" - when he developed the geometric theory of automorphic functions, combining Galois and

Riemann ideas - “was played by pictures” related to “transformation groups (linear fractional transformations of a complex variable)”. [13]

Klein’s hand-drawn diagram [14] related to elliptic modular functions is the same two-dimensional figure of four subspaces that we have seen forming the cobordant nucleus shared by the two intersecting fields.

The left and right transversal subspaces display a combination of half convex and half concave curvatures. The top vertical subspace presents a concave curvature, and the vertical inverted subspace exhibits a double convex curvature.

3. Conceptual translation to a quantum mechanics model

Once this model has been presented in a general way, we will try to describe it in terms of an atomic nucleus using a hypothetical approach that follows the symmetry or antisymmetry implicit in the Pauli exclusion principle.

The composition of the atomic antisymmetric nucleus will depend on the specific moment of the system’s evolution. It may consist of a proton, a positron and a neutrino, or an antiproton, an electron, and an antineutrino.

3.1 Antisymmetric system, the left intersecting field expands while the right one contracts **(A2)**

- The right contracting transversal subspace will represent a proton.
- The left expanding transversal subspace will represent a neutrino.
- The vertical subspace moving toward the right will represent a positron.

3.2 Antisymmetric system, the left intersecting field contracts while the right one expands **(A4)**

- The right contracting proton will expand, becoming a right expanding antineutrino.
- The left expanding neutrino will contract, becoming a left-handed contracting antiproton.
- The vertical positron will move toward the left, becoming an electron.

Fig. 13 visually represents the limit states of the evolution of the antisymmetric system; however, it does not reflect the moment when the top vertical subfield passes through the central axis, which is the reference center of symmetry of the system, carrying a neutral charge.

It is considered neutral because it is placed in the location used to distinguish between positive or negative: from that central point to the right the charge will be positive, and from that point to the left it will be negative.

This neutrality will occur during the intermediate expansion or contraction of the intersecting fields. In that case, the proton (or antiproton) transversal subfield, and the neutrino (or antineutrino) transversal subfield will show an isomorphic shape and their positive and negative charges will be in compensation. It may be at that moment when the notion of neutron and antineutron arises.

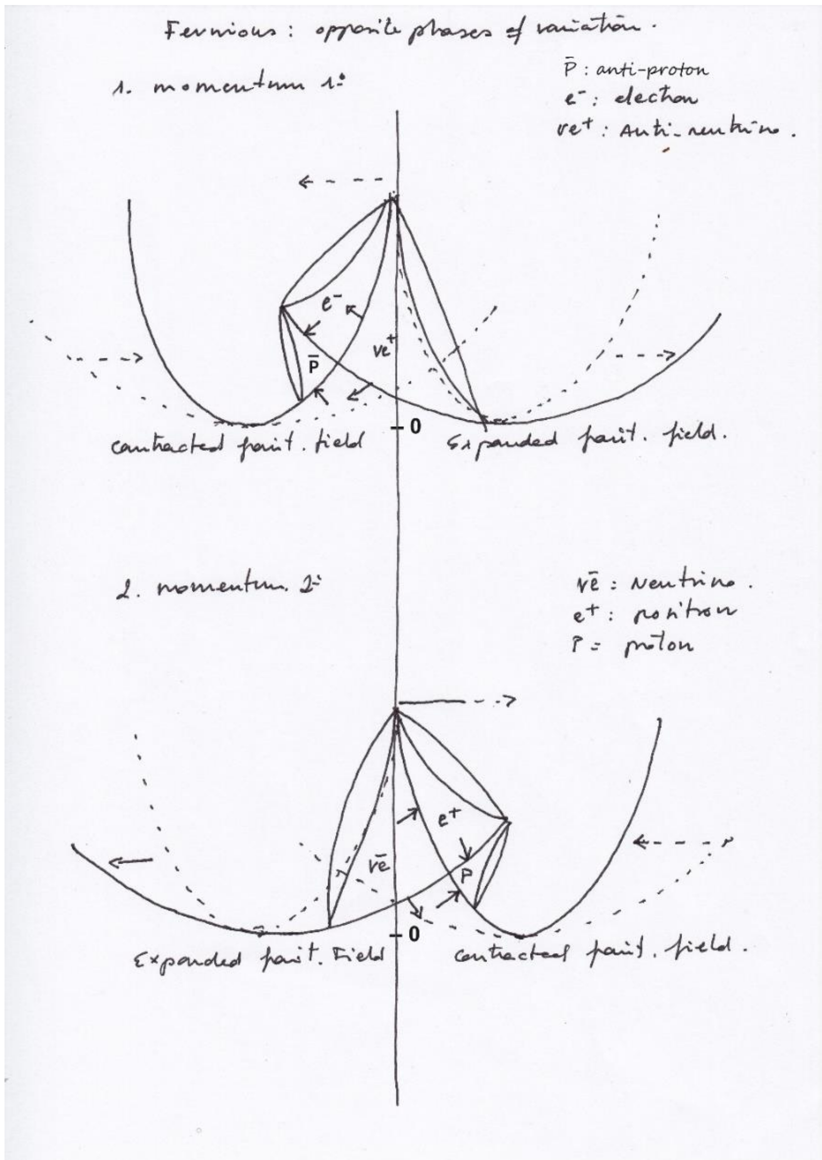

Fig. 13. Antisymmetric system at moment 1 (related to A2) and moment 2 (related to A4)

On the other hand, Fig. 13 diagram shows how the right-handed proton at moment A2 will decay, being virtually embedded in a right-handed antineutrino at moment A4, both in the right side of the mirror system.

Simultaneously, in the left side of the antisymmetric system an antiproton and an electron arise.

Later, the left-handed antiproton of A4 will decay into a left-handed neutrino at A2, while in the right side of the mirror system a proton and a positron will arise.

Proton and antiproton, and neutrino and antineutrino, will be Dirac antiparticles at different times.

Positron and electron are the same subfield, acting at different times as their own mirror reflection Majorana [15] antimatter.

The existence of an electron and a positron in the same atom, also known as "positronium" [16], was predicted by Dirac in 1928. However, positronium was formulated as an exotic atom with no proton in its nucleus.

The coexistence of proton and antiproton in the same atom is currently accepted as an exotic structure called "protonium" [17] with no electrons nor positrons.

In the dual atomic model, when it comes to the transversal subfields of the antisymmetric nucleus, isometric matter and antimatter are mutually exclusive at the same time, but they coexist as the chiral antisymmetric – with and advanced or delayed time phase – reflection of each other at the same moment, and as the chiral symmetric reflection of each other at different moments.

All the subfields in the antisymmetric system are fermions with noninteger spin, represented by the commuted eigenvector, being ruled by the Pauli exclusion principle. In that regard, they should adhere to Fermi-Dirac statistics, although the dual atomic nucleus is a causal model that can be described without using probability.

Additionally, in that same context, considering an antisymmetric Schrödinger's cat [18] as a figurative example, it could be said that the right alive contracting cat will be the delayed reflection of the left dead expanding cat, and vice versa.

It can be discussed whether they are the future or the passed reflection of each other, but that will only be a way to speak. There will not be a single alive and dead cat, but two identical cats with opposite states and positions. Their simultaneous states of being "alive" and "dead" can be considered "superposed" but in the context of their mirror antisymmetry.

It is visually represented in Fig.14:

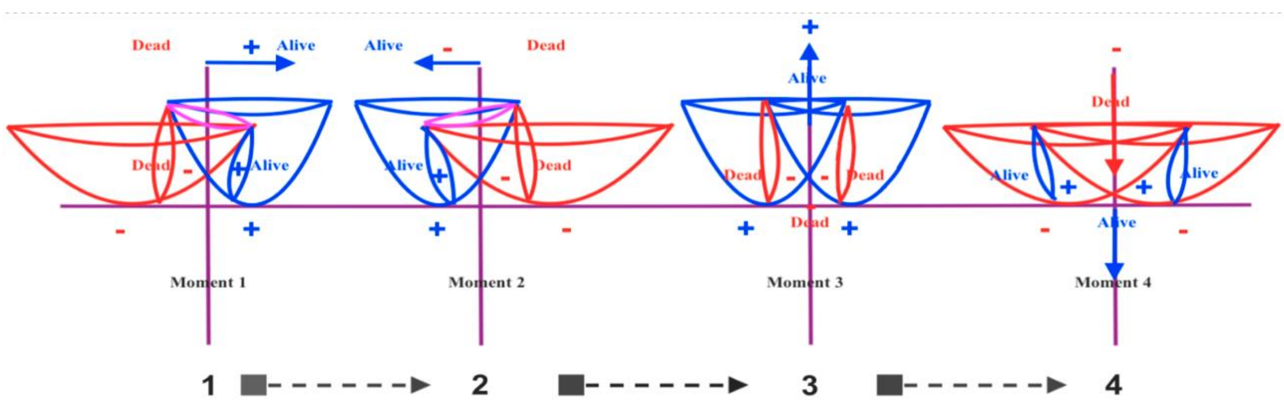

Fig. 14. Dead and/or alive Schrodinger cats in mirror antisymmetric and symmetric systems

3.3 Symmetric system, when the left and right intersecting fields contract (A1)

- The right and left expanding transversal subspaces represent a right-handed positive and a left-handed negative gluon.
- The top vertical ascending subspace that contracts receiving a double force of compression will be the electromagnetic subfield that emits a photon while pushing upward.
- The inverted bottom vertical subspace at the convex side of the system represents the dark decay of a previous dark antiphoton.

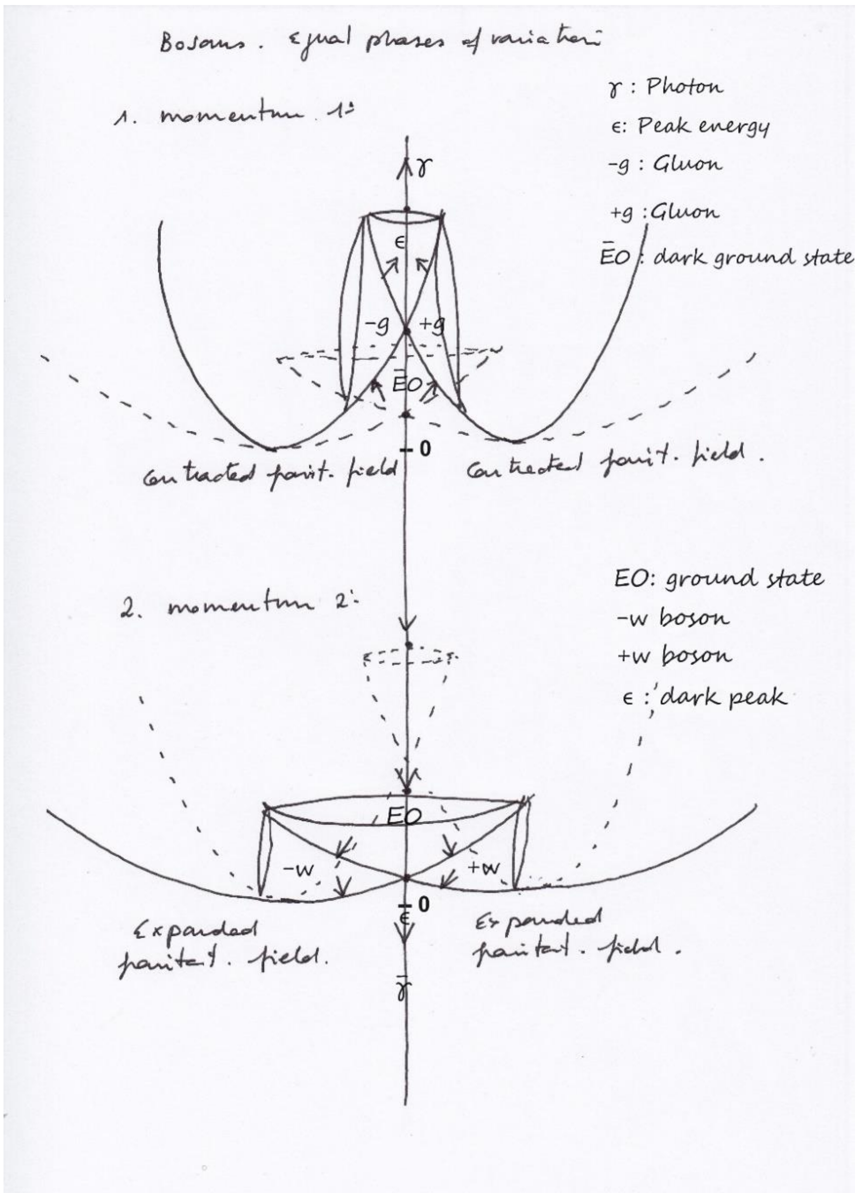

Fig. 15. Antisymmetric system at moment 1 (related to A2) and moment 2 (related to A4)

3.4 Symmetric system, when the left and right intersecting fields expand (A3)

- The right and left expanding transverse subspaces may represent -W and +W bosons.
- The top vertical descending subspace will be the electromagnetic subfield losing its previous energy, after having emitted a photon.
- The bottom vertical subspace at the convex side of the system is the dark anti electromagnetic subfield that emits a dark antiphoton.

It can be visually observed in Fig. 15 that the left and right transversal subspaces will be mirror symmetric antimatters at the same time, being bosons not ruled by the Pauli exclusion principle. They should then obey the Fermi-Dirac statistics.

However, the photon and the dark antiphoton – or the vertical subfield from which they emerge – are mutually exclusive. Therefore, they are governed by the Pauli exclusion principle, even though they have an integer spin represented by the two converging eigenvectors.

The identity of the symmetric transversal subfields, labeled before as “W bosons” and “gluons” requires further clarification.

Each of those subfields receives a bottom inward pushing force and a top outward decompression – in the strong interaction – or a top inward pushing force and a bottom outward decompression – in the weak interaction.

In the strong interaction, the magnitudes of the pushing forces caused by contracting or expanding intersecting fields will be different, because the contracting field exhibits a higher density, intensifying the propulsive force caused by the displacement of its negative curvature.

The vertical photonic subfield receives an inward double pushing force from right to left and from left to right caused by the displacement of the negative curvatures of the intersecting fields.

These pushing forces are the same as those that decompress the transversal subfields – labeled as gluons – at that moment. The emitted photon would have a double helix spin.

The pushing forces received at different moments by the moving right positron and the moving left electron in the antisymmetric system, now converge simultaneously in the photonic subfield.

From the perspective of this model, the transversal subspaces are the same topological subfields that contract when the intersecting fields expand in the weak interaction or expand when the intersecting fields contract in the strong interaction.

The strong and weak interactions, then, are related by the same mechanism. And the mirror transversal subfields that mediate the strong and weak interactions are the same topological subspaces that are transformed through time.

3.5 Supersymmetry

The model is N=1 because it relates in a supersymmetric way, through time, each fermionic subfield of the antisymmetric system with a bosonic subfield of the symmetric system.

In that sense:

- The fermionic electron-positron subfield will be the superpartner of the bosonic vertical subfield that emits the photon when ascending.
- The fermionic proton-antineutrino subfield, and the fermionic antiproton-neutrino subfields will be the superpartners of the symmetric transversal right and left subfields respectively, when they contract or expand.

In that way, the symmetry of the system is preserved through time. The modular Hamiltonian of the system also remains invariant through time.

3.6 Gravity and electromagnetism

On the other hand, the curvatures of the intersecting fields in this model are considered gravitational in both the symmetric and the antisymmetric systems. This implies that gravitational fields fluctuate or vibrate.

In this model, the electromagnetic charges are considered to be the pushing forces caused by the displacements of the subfields of the nucleus. These displacements are generated by the expansion or contraction of the intersecting gravitational fields that form the nuclear system, while expanding or contracting. The mass and energy of the nuclear subfields are also determined by the pushing forces derived from the variation of the intersecting gravitational fields.

In a different way, pushing gravity was already considered since Newton's time by Fatio and later Le Sage [19] [20] and others but was abandoned at the beginning of the XX century since Michelson and Morley demonstrated the nonexistence of a required ether.

However, today it is generally accepted that the Higgs field permeates the whole universe, and that the Higgs mechanism confers mass to the particles, by means of the Higgs bosons, which are the force carrier particles that represent the excitations of the Higgs field.

In that context, the variations in the intersecting gravitational fields may be considered "gauge bosons", and the dynamics of the intersecting system may be considered a Higgs mechanism.

3.7 Dirac spinors

A Dirac spinor is a group of four complex vectors that provide information about left or right handedness and about up or down orientation in a 4-dimensional space with 1 time dimension.

Using that formalism, it can be said that in the complete description of the dual atomic model four spinors are needed, two spinors for the fermionic system and two for the bosonic system.

Regarding the antisymmetric fermionic system of matrix A2, the magnetic spinor will be formed by the four right-handed eigenvectors. In that case, the ½ spin of the mirror reflection left and right transversal subfields or fermionic particles is formed by the right-handed down eigenvectors, which represent the partial conjugation of matrix A1 and a fractional number of derivatives. The state of each transversal subfield is determined by pairs of up and down eigenvectors, as explained before.

The dual atomic model uses two-time dimensions to describe the different phases of the antisymmetric dual nucleus. The dynamics of the spinor related to A4 can be interpreted as a time reverse direction with respect to A2. The four eigenvectors of matrix A4 represent the left-handed spinor, which is the negative reflection of the spinor of A2.

The four eigenvectors of matrix A1 form a bosonic spinor: two right-handed up eigenvectors and two left-handed up eigenvectors.

The negative reflection bosonic spinor of matrix A3 will be formed by two left-handed down eigenvectors and two right-handed down eigenvectors, combined in the ways previously described.

3.8 Supersymmetric quarks

The physical pushing forces created by the displacement of the intersecting fields when contracting or expanding, previously represented as eigenvectors, may be interpreted as "quarks" in the QCD terminology.

When a 90-degree rotation operator is applied, only two quarks change signs and the symmetric system becomes antisymmetric, and vice versa.

This cyclic invariance of the eigenvector quarks explains that their symmetry is preserved through time in a supersymmetric way.
Fig. 16 shows how the supersymmetric fermionic and bosonic "quarks" are transformed through time by periodically changing their sign by pairs.

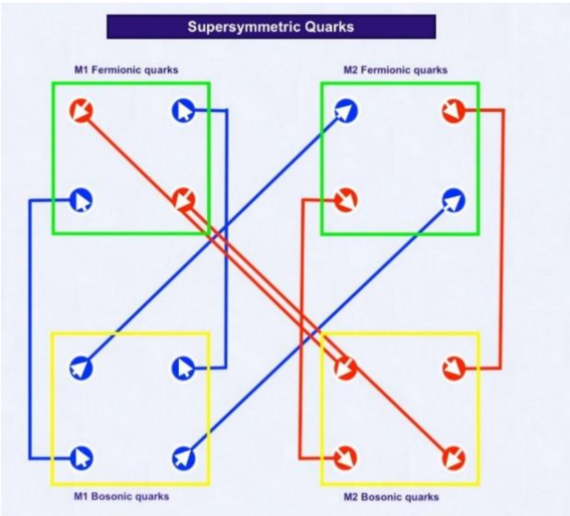

Fig. 16. Supersymmetric bosonic and fermionic quarks

3.9 Spatial topology

The two possible signs of the unitary eigenvectors may be related to the Hilbert space of dimension 2 we use a simplification of the higher dimensional system.

The space we have described for the bosonic symmetric and the fermionic antisymmetric systems can be considered a Minkowski space of 4 coordinates for two different frames of reference: x, y, z, t – for a real frame of reference – and x', y', z', t' – for a complex frame of reference whose coordinates are rotated 45 degrees relative to the real ones.

The 45-degree rotation of the coordinate system implies a partial conjugation that introduces the antisymmetry in the system, as previously noted.

A different time dimension for the bosonic and fermionic systems is necessary to describe their different dynamics, either because the subfields have an opposite phase between them, introducing a time delay in their mutual reflection, or because the subfields have an opposite phase with respect to the vibrations of the intersecting fields.

The dual atomic model suggests that the bosonic system is also antisymmetric in regard to the different phases of the transversal subfields with respect to the vertical subfields, which follow the phase of the intersecting fields.

The introduced model can be described in terms of moduli spaces of a Higgs bundle. Higgs bundles [21] were introduced by Nigel Hitchin. In the present model, the two intersecting fields would represent a Riemann geometry system.

The vertical subspace (and its mirror inverted counterpart) would represent the Higgs field that mediates between the left and right transversal subspaces, giving them their masses.

Both the left and right transversal subspaces, together with the intermediate Higgs field, form the moduli space of Higgs bundles.

The transversal subspaces represent the boundary components of the moduli space, while the Higgs field acts as a bridge between them. Those boundary components are frequently represented by two transversal tori.

Quantum mechanics has been developed mainly in an abstract mathematical way with no visual spatial references. However, the tori geometry is generally used in different ways by many theories to aid visualization and simplify calculations.

The intersecting spaces model can also be thought in terms of the topology of a two genus [22] torus, considering the outer positive and the inner negative curvatures of the torus as the simultaneous representation of the expanding or contracting moments of the vibrating fields when looking at them from above, in an orthographic projection represented in Figs. 17 and 18.

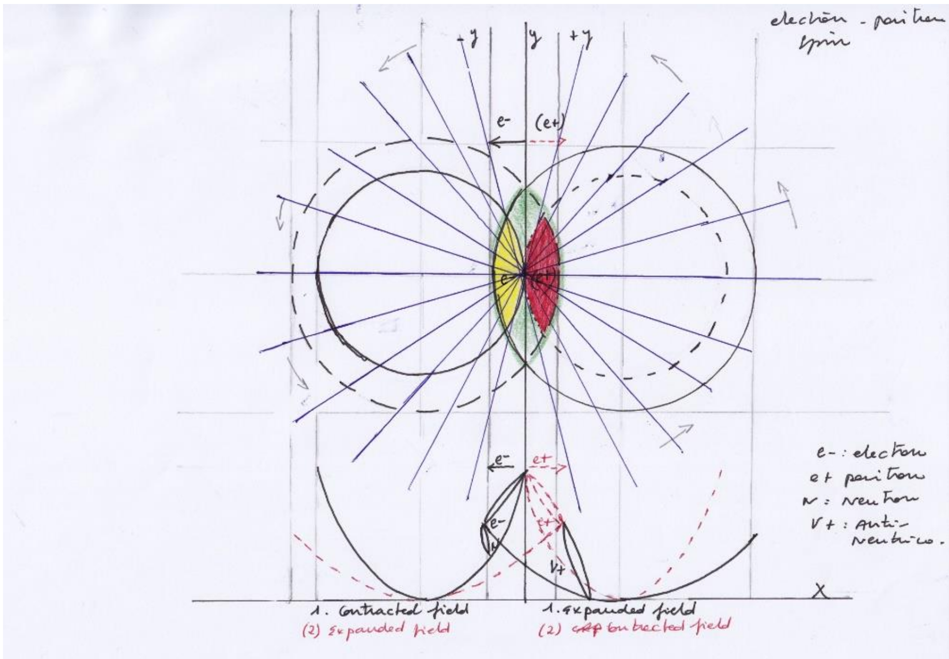

Fig. 17. Two genus torus projection of the antisymmetric system

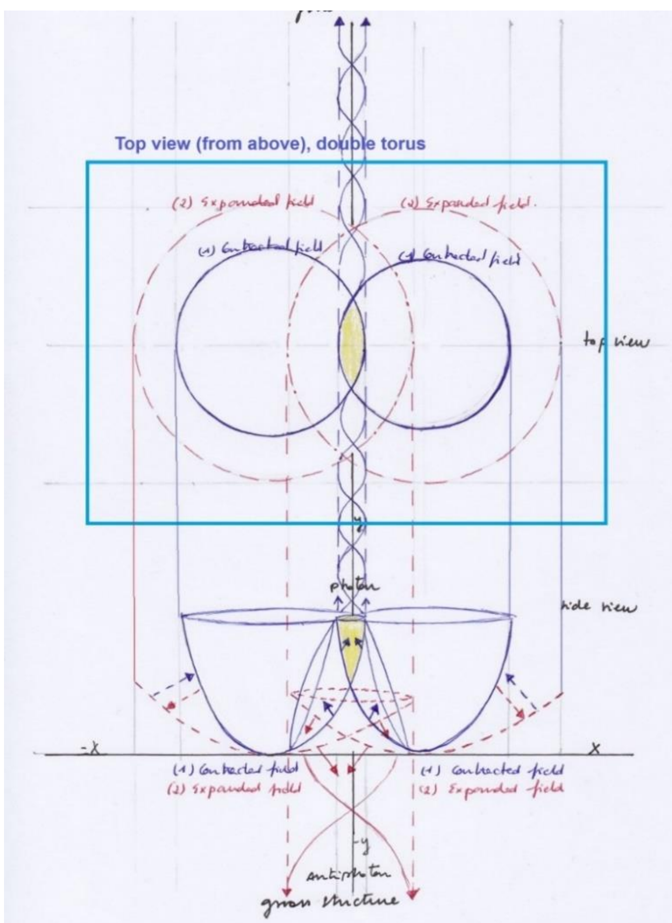

Fig. 18. Two genus torus projection of the symmetric system

The symmetric and antisymmetric subfields can be described as cobordant [23] subspaces. The vertical subspaces share borders with the left and right transversal subspaces, and they all share borders with the two intersecting spaces. These borders can be thought of as unidimensional lines described by the curvatures of the intersecting fields.

The geometry of the intersecting fields model can be also related to Hyperkahler and Kummer quartic surfaces [24].

On the other hand, the shape of the nuclear transversal subfields, on the symmetric or antisymmetric diagrams, resembles the shapes drawn by Lobachevsky [25] when explaining his imaginary geometry.

The topology of the nuclear system may also be described in terms of two double oscillators.

Note that the inner curvature of both transversal subfields is half negative (curved inwards) and half positive (curved outwards). The top vertical subfield in the symmetric and antisymmetric system will have an inner concave curvature, while the bottom inverted vertical subfield will have a double region of positive or convex curvatures.

The topology of the nuclear system may also be described in terms of two double oscillators, four coupled vortices.

The curvatures of the intersecting fields can also be described in terms of longitudinal waves.

3.10 Special Relativity

Different mathematical transformations, such as Fourier transforms and Wick rotations, are being used to make antisymmetric systems operationally symmetric.

Mathematical transformations will also be required to relate the two sets of coordinates associated with the fermionic and bosonic systems.

The main difficulty is that the YX coordinates and the diagonal axis that divides the complex plane are referenced by different metrics. One point at Y cannot be rotated to $X + iY$ without increasing the spatial distances, and so the time needed to cover them, as it happens in Special relativity.

The same happens with the hypotenuse of a right triangle when trying to measure it with the reference metric of the legs.

The infinite decimals of irrationality arise when two different frames of reference are being measured with the same gauge.

The metric issues between the two reference frames can be managed by adding additional spatial dimensions to separately describe each space-time system or subsystem (in the case of the transversal subfields).

3.11 Möbius transformations

However, a type of mathematical transformation is needed to relate the coordinates of the two reference frames. This is the subject of Lorentz transformations in special relativity, which are a type of Möbius transformation [26].

Möbius transformations can be used to project the Y and X coordinates to the imaginary points at X' and Y', virtually removing the complex plane while preserving the angles.

It can be superposed on a same picture the real and complex planes of the symmetric and antisymmetric systems, representing the result of convolving the complex and conjugate functions, as if the four partial conjugations were taking place at the same time.

The complex plane can therefore be treated as a real plane.

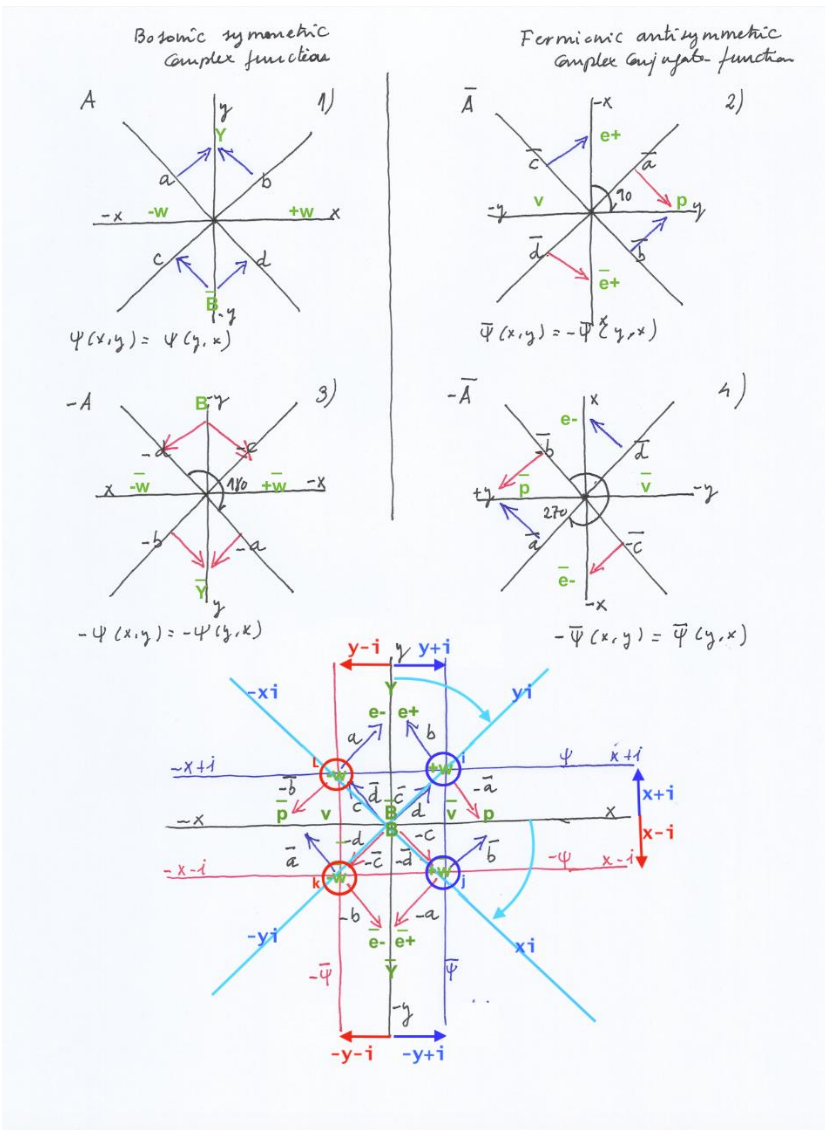

Fig. 19. Superposition of spaces and times evolution.

Fig. 19 simultaneously shows all the possible states that the vectors get through time, including the left and right displacements related to the fermionic nucleus (from a central axis Y towards a projected +Y’ or −Y’), and the upward and downward displacements related to the bosonic nucleus (from a central axis X toward a projected +X’ or −X’), which can be interpreted as a nuclear precession.

By means of the vectors in the diagram, the symmetry of both bosonic and fermionic systems is shown to be reached over time. The supersymmetric topological transformation of the nuclear subfields, which may take place by means of phase synchronization, the rotation of the whole system, or both, is a spatial and temporal gauge transformation.

3.12 The emergence of imaginary time

From the perspective of a dual atomic model, time – as a necessary reference for measuring variation – is a notion that cannot be separated from space in regard to periodically varying folds.

However, between the moment of the highest expansion or contraction, until the opposite contraction or expansion starts, there will be a period of no spatial variation that may be interpreted as no time.

In the case of a composite manifold with different phases of variation, an additional dimension for the emerging time is needed.

Considering time as a real axis tY in the symmetric modular group A1 A3, the partial conjugation operated by a 90-degree rotation introduces in the system a new time coordinate ti, which is purely imaginary and causes the emergence of the spatial-time antisymmetry in the modular group A2 A4.

Using the previously mentioned Tomita Takesaki terminology, the handed sides of the modular operator A2 (and the sides of its automorphic modular conjugate A4) become antisymmetric because half of the system will follow the real time tY, while the mirror side will follow the new imaginary time ti, creating a time-gap between both sides of the modular operator and, later, of the automorphic modular conjugate.

The imaginary time is not only a symbolic cartesian formalism, but also a mathematical way to describe the dynamics of the rotational system that needs a second time dimension to refer to the harmonic ti phase introduced in the modular group by partial conjugation.

Commuting the sign of half of the eigenvectors while leaving the other half unchanged implies a partial differentiation that can be interpreted as a fractional derivative or antiderivative, as we saw before. That fractional derivation creates the harmonic or fractional imaginary time and the fractional spins in the modular antisymmetric – anticommutative group.

Fractionality is responsible for the disruption of symmetry inherent to the commutative - or interchangeable - group, causing the antisymmetry associated with the anticommutative group.

4. Relations with other theoretical models

To conclude, we can relate the atomic model to other theories and developments, using a visual approach as well.

4.1 String theories

In the context of string theories, the border of the positive and negative curvatures of the intersecting fields, or of parts of them, may be seen as one-dimensional open or closed strings:

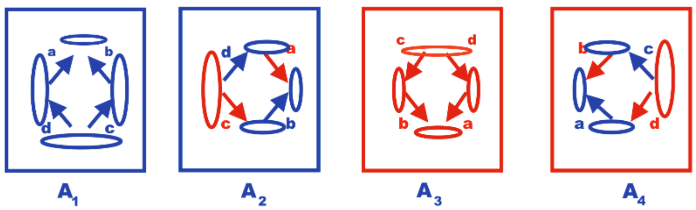

Fig. 22. Unidimensional closed strings

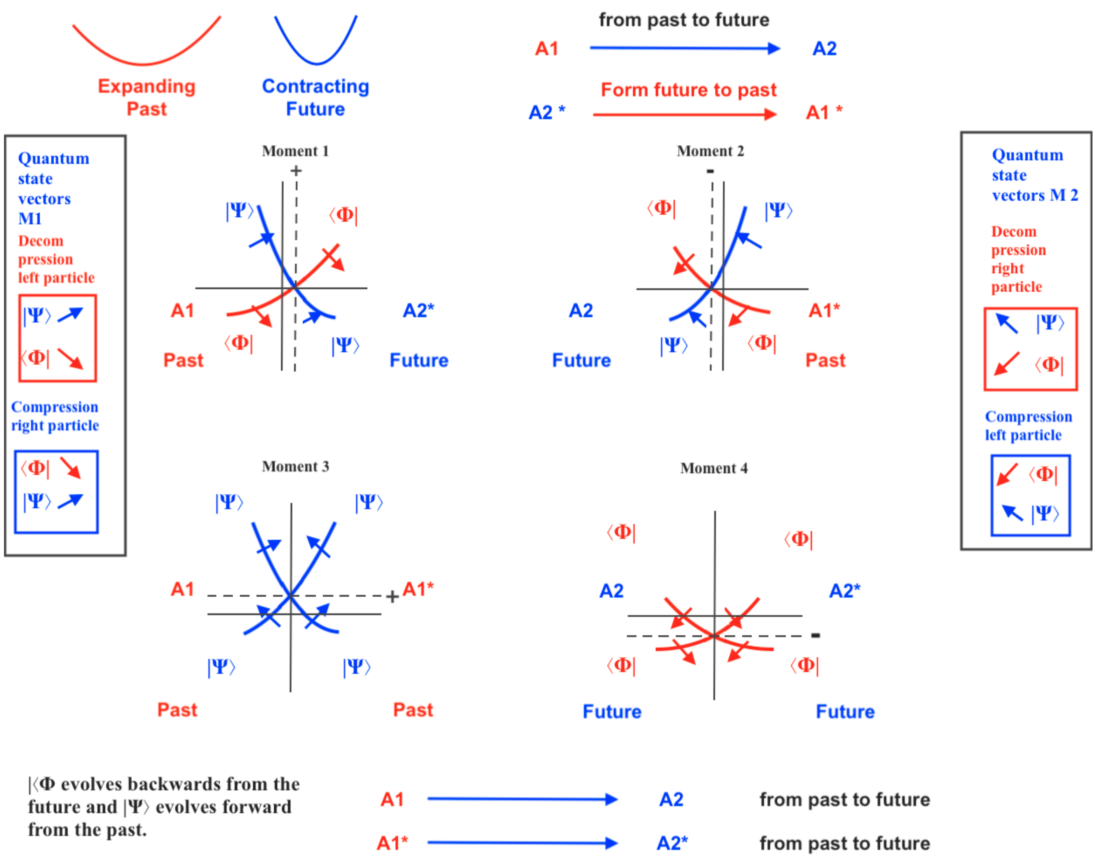

Fig. 20. Unidimensional open strings

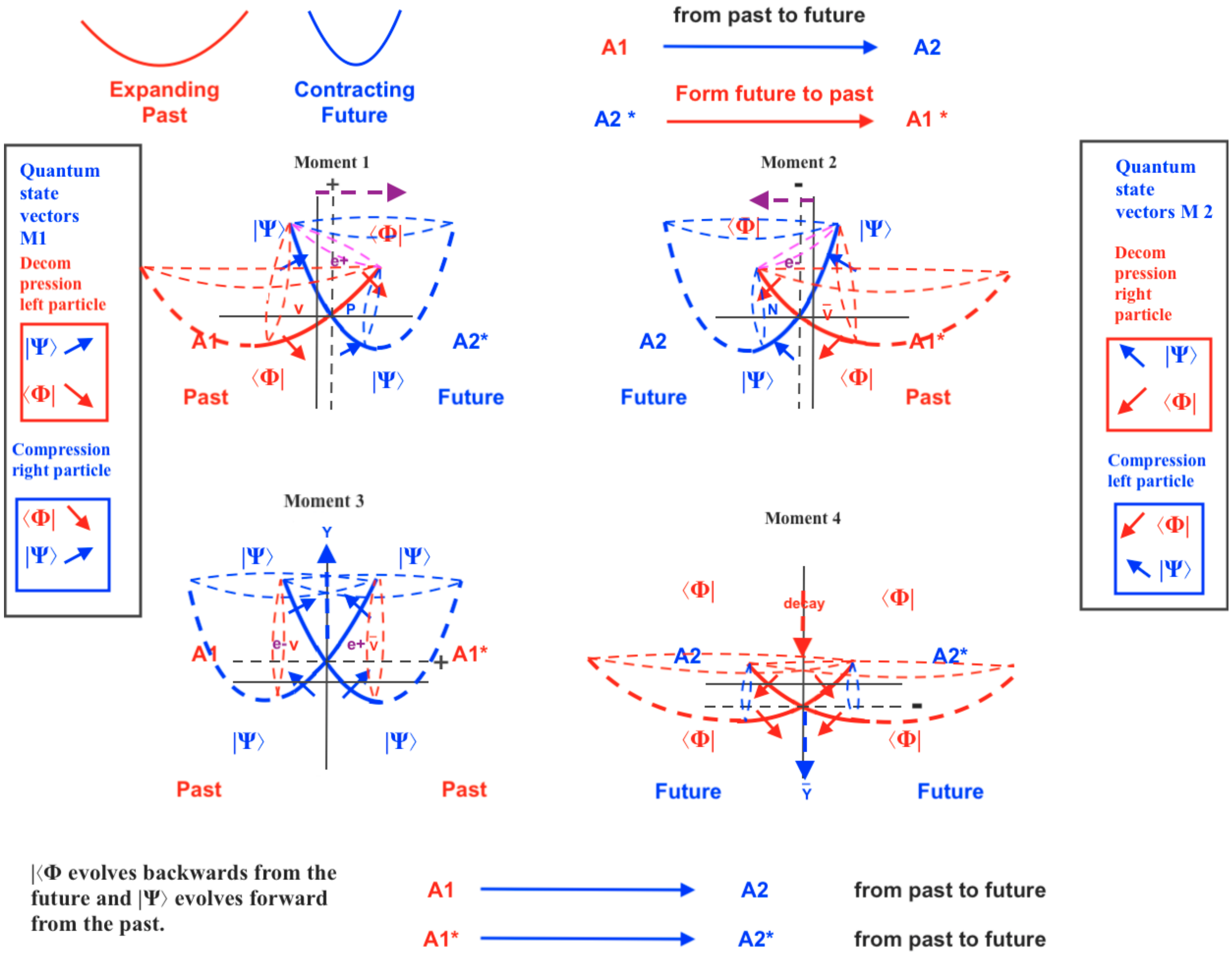

Fig. 21. Unidimensional strings embedded in the dual quantum field model

In string theory, mirror symmetry emerges from the notion of T-duality [27], that relates the spaces described by Type IIA and type IIB strings theories.

In Type IIA string theory, the strings can move freely in the Calabi-Yau [28] space with a larger radius, while in type IIB string theory, the strings are confined to the boundaries of the transverse space of shorter radius.

T-duality relates these two different types of larger and smaller transversal spaces by means of a type of inversion that exchanges the roles of the large and small radii transverse spaces.

In the context of the dual fields model, the Calabi-Yau spaces of smaller or larger radius are considered equivalent to the transverse higher dimensional contracting or expanding subspaces, that are mapped to each other in a mirror symmetric way by means of their topological transformation through time, as described before in the antisymmetric system.

In the fields model, the transverse subspaces periodically change their role in the antisymmetric system becoming the negative reflection of each other at different times. This type of mirror symmetry at different times may be related to the SYZ conjecture [29], which states that there exists a special type of Calabi-Yau manifold called a mirror manifold that is related to another Calabi-Yau manifold by a T-duality transformation.

The elliptic orbits inside of the transversal subfields, caused by their periodical expansion and contraction, can be visually related to the notion of elliptic fibrations used in String theories.

They are represented in Figs. 23 and 24.

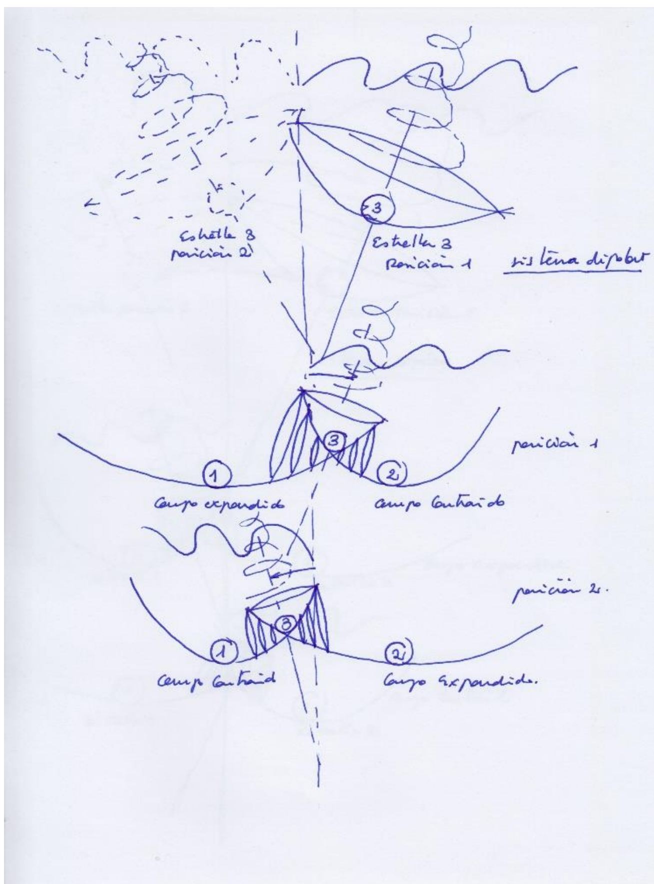

Fig. 23 Elliptic fibrations in antisymmetric system

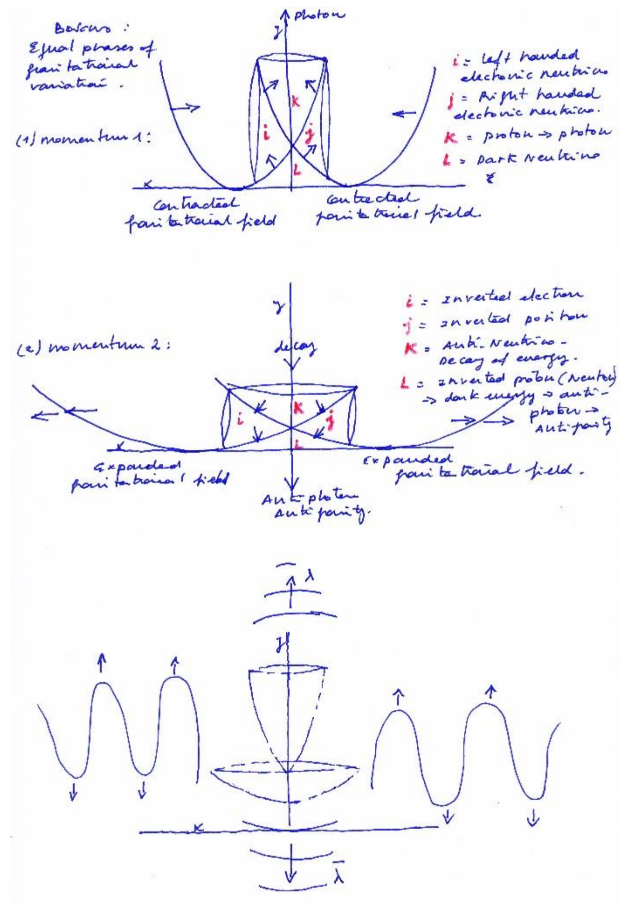

Fig. 24. Elliptic fibrations in the symmetric system

As well in the context of string theories, the intersecting fields that interact to form the nucleus of subfields may be related to the positive curvature of de Sitter vacuum spaces, when expand causing an outward pushing force, or to the negative curvature of anti de Sitter [30] vacuum spaces, when they contract causing an inward pushing force.

The symmetric and antisymmetric systems may also be related to the Ramond-Ramond or the Kalb-Ramond fields. The Ramond-Ramond [31] fields are antisymmetric tensor fields with two spacetime indices, which is a mathematical way to refer to two fields in a dual system.

4.2 Many worlds interpretation

Everett's many worlds interpretation [32] of quantum mechanics proposes multiple worlds that coexist simultaneously.

Everrett's multiple worlds are independent of each other and do not interact, although there is also an interacting version of the Many Worlds interpretation by H. Wiseman [33].

In the MWI, each world represents a possible state of a particle. In this sense – despite the differences – the "intersecting worlds" model introduced in this paper seems conceptually consistent with the idea of multiverses with different states, in the case of the antisymmetric system, or even with equal states in the case of the symmetric system.

However, the parallel universes in the dual model are conceptualized in terms of the mirror reflection matter and antimatter existing in a nuclear manifold, instead of considering the states of matter or antimatter existing in independent and unrelated folds.

The notion of multiverse is assumed in the present model.

4.3 Wave pilot interpretation

In 1952 David Bohm, based on de Broglie's work, proposed the wave pilot interpretation of quantum mechanics. Previously, the idea that photons or electrons may be guided in some way by a pilot electromagnetic field or by a pilot wavefunction had been considered by Einstein and Born respectively. [34]

The present model may be interpreted as a two-wave pilot theory, as the displacements of the photonic subfield in the symmetric system and the electron-positron subfield in the antisymmetric system are guided or piloted by the displacements of both intersecting fields while contracting or expanding with the same or opposite phases.

The intersecting fields and subfields can be characterized as longitudinal waves.

The "hidden variables" in the dual atomic model are related to the composite spatial structure given by the intersection. Being a local model, whatever action on the left or right side of the system will have immediate effect on the whole shared nucleus.

Action at a distance will have the limit given by the intertwined topology of the dual system.

4.4 Hidden sectors and Hidden valley models

The hidden sector [35] or hidden valley is a hypothetical collection of quantum fields and their related particles that are not visible to us and are considered related to dark matter.

In the context of the dual matter atom of six folds we introduce in this paper, a hidden sector will be the field or subfield that cannot be directly observed from inside of another field or subfield of the composite atomic manifold.

In that sense, an observer placed inside the left intersecting field or in the left transversal subfield will not be able to directly detect the right intersecting field nor its related transversal subfield. An observer placed in the vertical subfield with negative curvature will not directly detect the inverse vertical subfield with positive curvature.

In the context of the mirror symmetric or antisymmetric dual system, the “visibility” or invisibility of the “dark” mass or energy will be relative to the position of the observer.

4.5 Transactional interpretation

The presented model can also be expressed in terms of emitter-absorber transactional models that correlate advanced and retarded waves. The opposite states of the nuclear subfields can be viewed as an interchange of energy that occurs through a transactional “handshaking” [36],

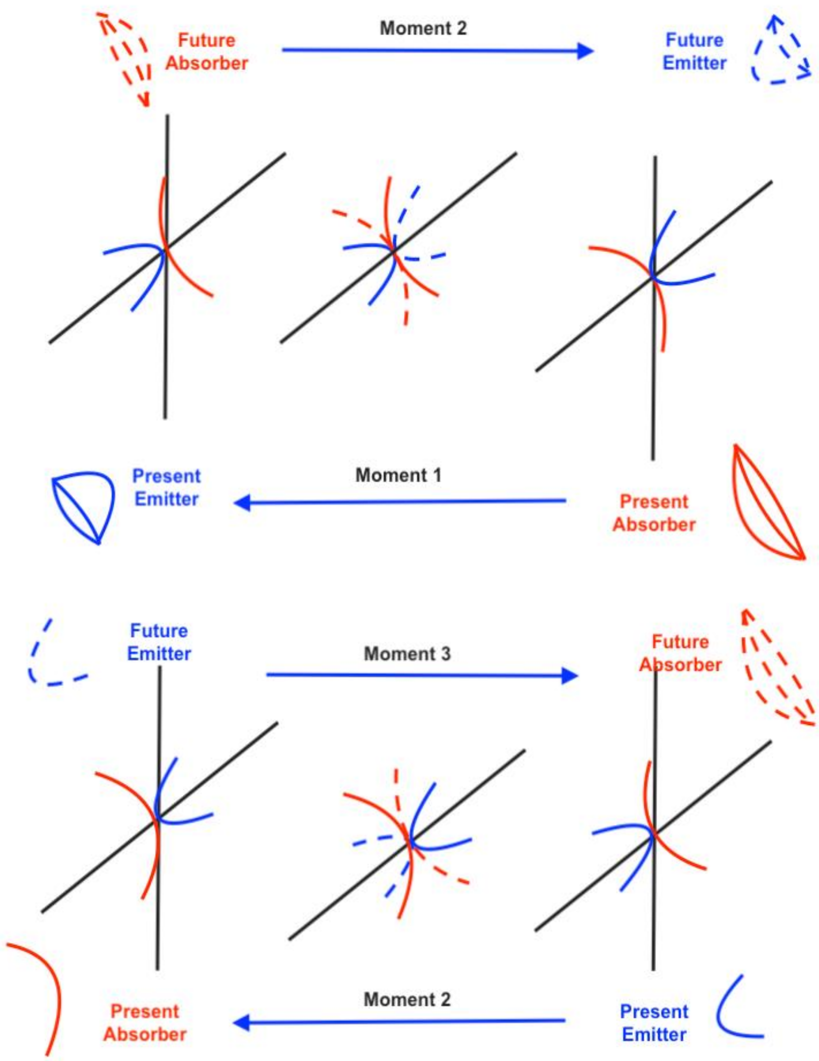

Fig. 25. Transactional handshaking

4.6 Dirac and Weyl semimetals

Weyl semimetals [37] are considered exotic phases of matter characterized by the presence of Weyl fermions, which are considered ½ spin quasiparticles with chiral handedness and no mass or charge. They were proposed by H. Weyl in 1929.

In that context, it is frequent to represent the Weyl fermion [38] as the subcone formed by the intersection of two cones.

This type of geometry is closely related to the topology of the dual atomic model introduced in this article. In the context of the dual atomic model, the subcone would the vertical subfield formed by the intersection of a contracting and an expanding field that vary with opposite phases forming an antisymmetric system. The vertical subfield is identified as an electron when it moves left toward the side of the intersecting field that contracts, or as a positron when it moves right a moment later.

However, as the Weyl fermion does not have mass and has a fixed chirality that breaks the CP party that links matter and antimatter, it can also be related to the left or right transverse subfields that this model identifies as an expanding and almost massless neutrino or antineutrino.

The CP parity of the handed neutrino is saved through time, as its mirror reflection counterpart will exist in a future or past time, so to speak, when the left transversal subfield expands at the left side of the system acting as a neutrino, the right transversal subfield will contract acting as a proton, and the vertical subfield will move towards the right acting as a positron. A moment later, when the left transversal subfield contracts becoming an antiproton, the right transversal subfield expands acting as an antineutrino, and the vertical subfield moves towards the left acting as an electron.

In that sense, the massless transversal "Weyl fermion" would be related to the decay of the proton and antiproton, and to the arising of the electron or positron.

H. Weyl formulated his hypothetical massless fermion in 1929. The massless neutrino was proposed by Pauli in 1930.

4.7 Redox and acid-base reactions

The dynamics of the dual nucleus, in regard to the antisymmetric system, can be conceptualized as a type of reduction-oxidation (redox) reaction, where the right-handed and left-handed sides of the system interchange roles as oxidizing and reducing agents:

- In matrix $A2$, the right-hand side acts as an oxidizing agent that gains a positron, becoming reduced, while the left-hand side acts as a reducing agent that loses an electron, becoming oxidized.
- Conversely, in matrix $A4$, the right-handed side acts as a reducing agent that loses a positron, becoming oxidized, while the left-handed side acts as an oxidizing agent that gains an electron, becoming reduced.

The interplay of the left- and right-handed sides of the antisymmetric system can be conceptualized as well as an acid-base reaction between the two sides.

- In matrix $A2$, the left-handed side acts as an acid donor, transferring an antiproton to the right-handed side which acts as a base acceptor receiving a proton.

- In contrast, in matrix $A4$, the left-handed side acts as a base acceptor, receiving an antiproton from the right-handed side, which acts as an acid proton donor.

The reciprocal transfer of mass, energy, and charges between the two sides of the mirror antisymmetric system is a consequence of their different oscillatory phases.

4.8 Riemann-Silberstein vector

In the early 20th century, Ludwik Silberstein proposed a novel formulation of Maxwell's equations in which the electromagnetic field is represented by a complex vector, known as the Riemann-Silberstein vector [39]: $F = E + iB$

The electric field E constitutes the real part of the complex vector F, and the magnetic field B forms its imaginary part. This implies a 45-degree rotation of the magnetic field B and the introduction of a different time phase in the magnetic field.

The dual atomic model discussed in this article suggests that the new time phase affects only half of the magnetic system at matrix $A2$ moment and, later, the other half at matrix $A4$. This nonlinear evolution would imply that the interaction between the electric and magnetic fields cannot be described by simply adding the electric and magnetic fields together.

However, as was previously shown in Fig.16, it can be useful to represent the evolution of the symmetric (electric) and antisymmetric (magnetic) systems as simultaneously superimposed on the same complex space.

In the context of the dual nucleus, the upward and downward displacements of the vertical subfield of matrix $A1$ and $A3$ cause the electric field, and the rightwards and leftwards displacements of the vertical subfield of matrix $A2$ and $A3$ are the source of the magnetic field.

By rotating the complex plane 90 degrees four times, the quantized electric and magnetic behavior of the system can be described by the sequence `F = A1 + A2 + A3 + A4`.

`A1` and `A3` matrices can be interpreted as representing the electric source of the fields system, while matrices `A2` and `A3` represent its magnetic source.

`A2` represents the positive monopole, and `A4` represents the negative monopole moments of the dipole formed by `A2 + A4`.

The leftward and rightward displacements of the subfield that create the lateral pushing electric charge in the magnetic moments of `A2` and `A4` are caused by the introduction of a time phase shift in half of the system, making it antisymmetric.

The symmetry is restored in the electric moments of `A1` and `A4` when the time phases synchronize.

Fig. 26 visually relates the four eigenvector matrices with the diagrams of quantized EM waves symbolically represented in a sinusoidal way:

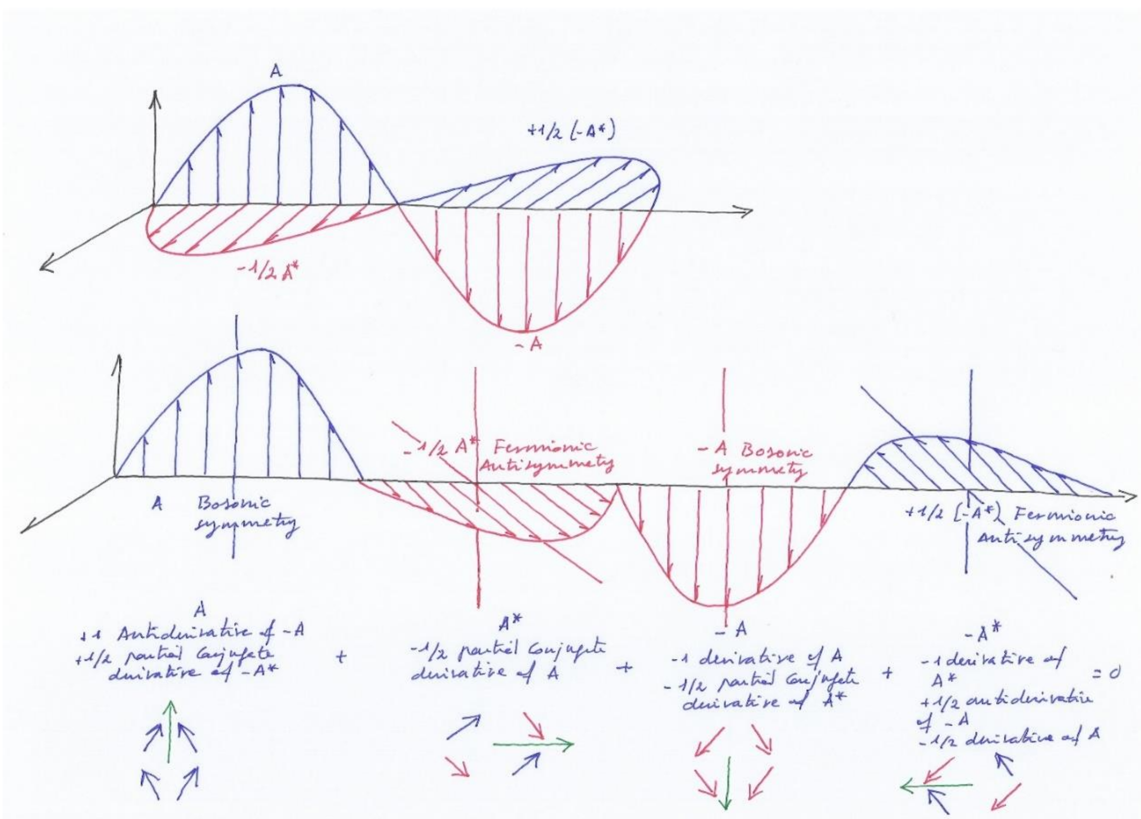

Fig. 26. Representation of the Fermionic magnetic and bosonic electric moments

4.9 Deterministic theories: Causal set and Cellular automata .

Several theories as the Causal set theory [40] formulated by Rafael Sorkin, or the Cellular automaton [41] interpretation of Quantum mechanics proposed by G. ‘t Hooft, look for a deterministic description of nature at the micro and macrocosmic level.

However, how causal discreteness arise from a classical continuous space time remains to be fully elucidated.

The geometry of the rotational dual atomic model may enrich those theories by providing a deterministic topological description for the smooth continuity hidden in the atomic discreteness, and for the discreteness that may be hidden in the continuity of space time when it forms a composite rotational manifold.

As it has been previously seen, it’s the periodic 90-degree rotation of the whole complex plane what introduces a quantum discontinuity in the classical continuous evolution of the dual system, interpolating the antisymmetric and the symmetric moments of the system.

That rotational emergence of the causal and smooth discreteness is graphically deduced from the evolution of the 2x2 complex rotational matrices of eigenvectors A1 to A4.

4.10 Astrophysics

Copernicus started to question the geocentric model because of its unexplained asymmetries, considering it "a monster" formed by unrelated members taken from different places. [42]

However, his heliocentric model lost its circular symmetry when Kepler realized, based on the more precise measures given by the invented telescopes, that planetary orbits were elliptic.

Although the Copernican simplicity remained, many unexplained asymmetries emerged: different planetary velocities, planetary motions that accelerate and decelerate, different orbital eccentricities, different inclinations, and even planets that rotate in opposite directions.

Several explanations have been proposed ad hoc, for example for the inverse rotation of Venus, and although the asymmetries of solar systems can be mathematically predicted and described they are not explained by means of a unique mechanism.

A single and invariant orbital field would not be enough to describe a composite manifold of intertwined spaces and subspaces varying with the same or opposite phases.

A system of intersecting universes that fluctuate with the same or opposite phases would produce a periodical big bang in the concave side of the symmetric system followed by a big silence when an inverse and dark (as directly undetected from the concave side) big bang occurs in the convex side.

The detected discrepancy in the measures of the rate at which the universe expands, known as the "Hubble tension" [43], could be related to the different pushing forces that are caused by the negative and positive curvatures of intersecting universes that periodically fluctuate.

In a rotational multiverse context, a new time phase will emerge in half of the system following a big bang, as one side continues to contract while the other side commences to expand.

4.11 Biophysics

The fields and subfields of the model can also be interpreted as fluctuating vortices. Some quantum theories, such as the Chern-Simons theory [44], consider vortices instead of fields.

The article titled "The Chern-Simons current in systems of DNA-RNA transcriptions" [45], provides an interesting diagram of a loop space acting as a Hopf fibration [46] over a DNA molecule [47]. This diagram bears resemblance to the diagrams about the inner orbits or elliptic fibrations of the transversal subfields previously mentioned.

On the other hand, it's currently known that the rotation of the spindle axis plays a fundamental role during cell division and that abnormal spindle rotations [48] can result in genetic abnormalities.

Considering a cell as a dual system, its inner rotational dynamics may present some similarities with the physics of the dual atomic model we have presented. Abnormal phase changes would affect the processes of cell division and differentiation, accelerating or decelerating their normal paths.

5. Model inconsistencies

This paper does not aim to provide a rigorous formulation of the dual atomic model it introduces. Its focus is to show the consistency and possibilities of a dual approach to the atomic nucleus.

5.1 The rotation of the whole system, which seems necessary to quantize the classical continuous fields and subfields of the intersecting system, does not seem to be enough to transform per se the physical properties and phase times of the subfields, synchronizing and desynchronizing the phases of the system.

Misinterpreting the physical meaning of the symbolic formalism given by the eigenvectors' directions seems to be a possibility in this model.

5.2 The model does not explain how the inner orbital motions of the transvers subfields are affected by the inner orbital motions of the intersecting fields when contracting or expanding.

The inward pushing forces of the contracting fields with their subsequent compression and orbital acceleration, and their outward pushing forces when expanding with their subsequent decompression and orbital deceleration, will affect the orbits of the transversal subfields, especially where their orthogonal convergence occurs forming a shared whirlpool or vortex.

That vortex could induce an epicycle-like trajectory of the inner orbital motions within the transversal subfields, which would require further explanation.

5.3 Theis paper does not discuss the cause of the periodical curvatures of the intersecting fields, although it relates the system to the Higgs mechanism in a gravitational way.

To describe the periodical curvatures in terms of spatial density and friction in a thermodynamic way a source of generating pushing force is needed.

6. RECAPITULATION AND FINAL REMARKS

The introduced topological quantum fields model is based on two intersecting fields whose curvatures periodically fluctuate in either the same or opposite phases. Their intersection forms a shared nucleus of two transversal and two vertical subfields that represent the particles and antiparticles of the dual nucleus shared by the system.

When the curvatures of the intersecting fields vary with opposite phases, the transversal subfields exhibit mirror symmetry at different times, as half of the system follows a delayed phase that implies the emergence of a purely imaginary time dimension; the vertical subfield will move left or right, towards the side of the intersecting fields that contracts, acting as its own antiparticle. When their phases synchronize, the transversal subfields become mirror symmetric at the same time and the vertical subfield will move upward or downward.

The dynamics of both symmetric and antisymmetric systems exhibit a classical continuity, like longitudinal waves, even in the case the intersecting fields synchronize and desynchronize periodically. However, considering a periodic 90-degree rotation of the whole system, a discrete discontinuity is introduced in its classical continuous evolution, breaking and restoring periodically its inner reflection symmetry.

That rotational discreteness can be deduced from a 2x2 complex rotational matrix of eigenvectors and their related matrices transformed by the operations of transposition, inversion, complex conjugation and reversion, related in a modular way to Sobolev interpolations and to Tomita-Takesaki theory.

The commutation of the sign of two of the four eigenvectors indicates the ½ or the 1 (as ½ + ½) order of derivation or antiderivation, and therefore the spin related to the state and spatial displacement of the subfields in the – periodically interpolated – antisymmetric and symmetric moments of the rotationally supersymmetric system.

The nuclear subfields can be interpreted as the subatomic particles of a dual nucleus formed by matter and mirror antimatter, describing the strong, weak and electromagnetic interactions. The 90-degree rotational transformations introduce a smooth quantum discontinuity, enabling the description of the curved space-time in discrete terms.

Superposition, entanglement, and the Pauli exclusion principle are interpreted in terms of mirror reflection matter and antimatter.

A supersymmetric non-probabilistic description of the whole rotational system requires the interpolation of the complex function that describes its bosonic symmetric states and the harmonic conjugate function that describes its fermionic antisymmetric states. The bosonic symmetric system acts as the electric moment of the rotational manifold, and the fermionic antisymmetric system acts as its harmonic magnetic moment.

While this work does not provide a fully developed and rigorously formulated model, the author hopes that its visual geometric and heuristically conceptual approach will inspire future advancements in research on the fundamentals of quantum mechanics and the physics beyond the standard model.

The author further suggests that the introduced dual fields model could provide valuable insights for further modeling advances in the dynamical behavior of abnormal cell division and differentiation, which has been his primary objective.

This work is dedicated to the memory of Magdalena.

7. Additional images

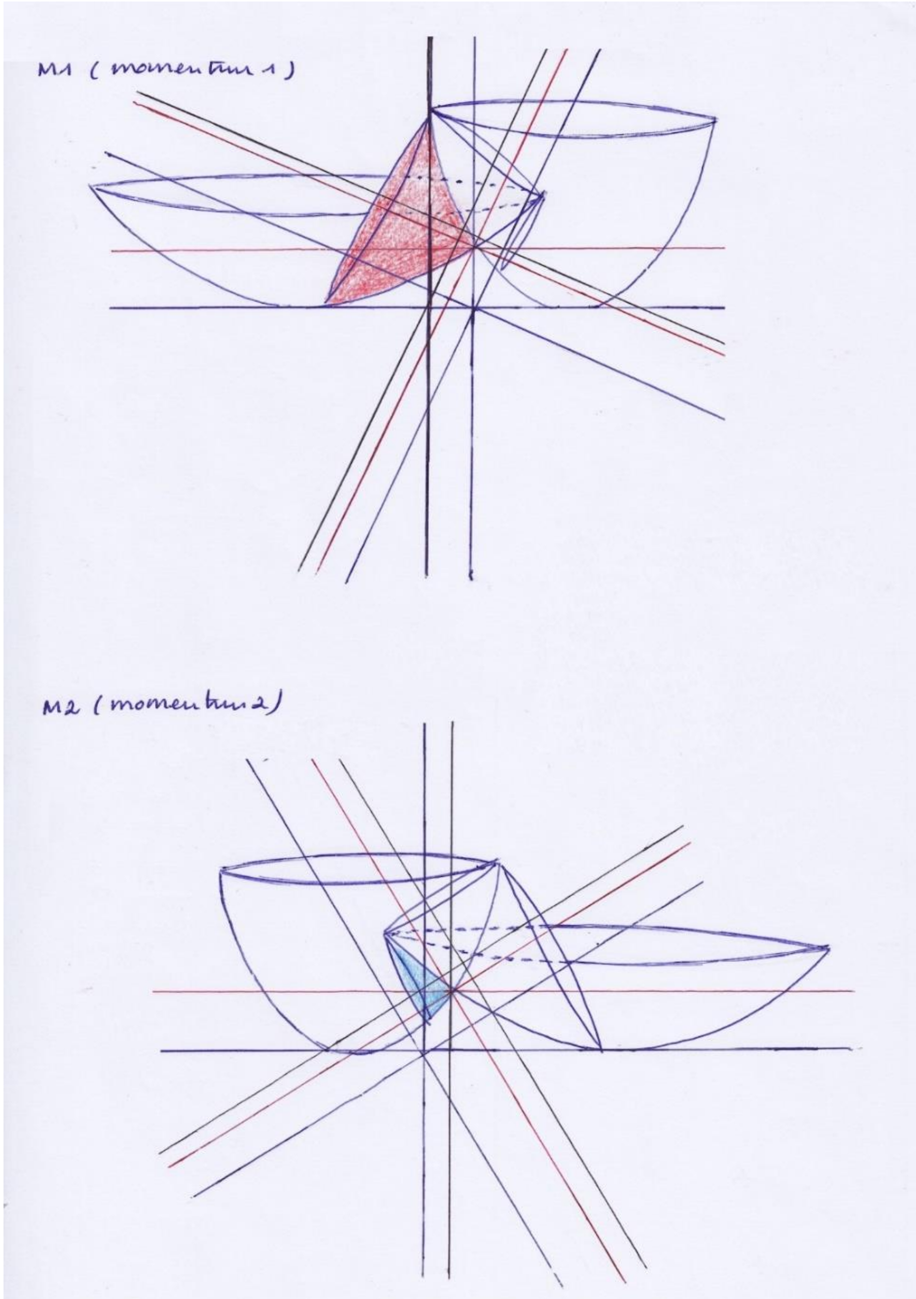

Fig. 27. Frames of reference, antisymmetric system

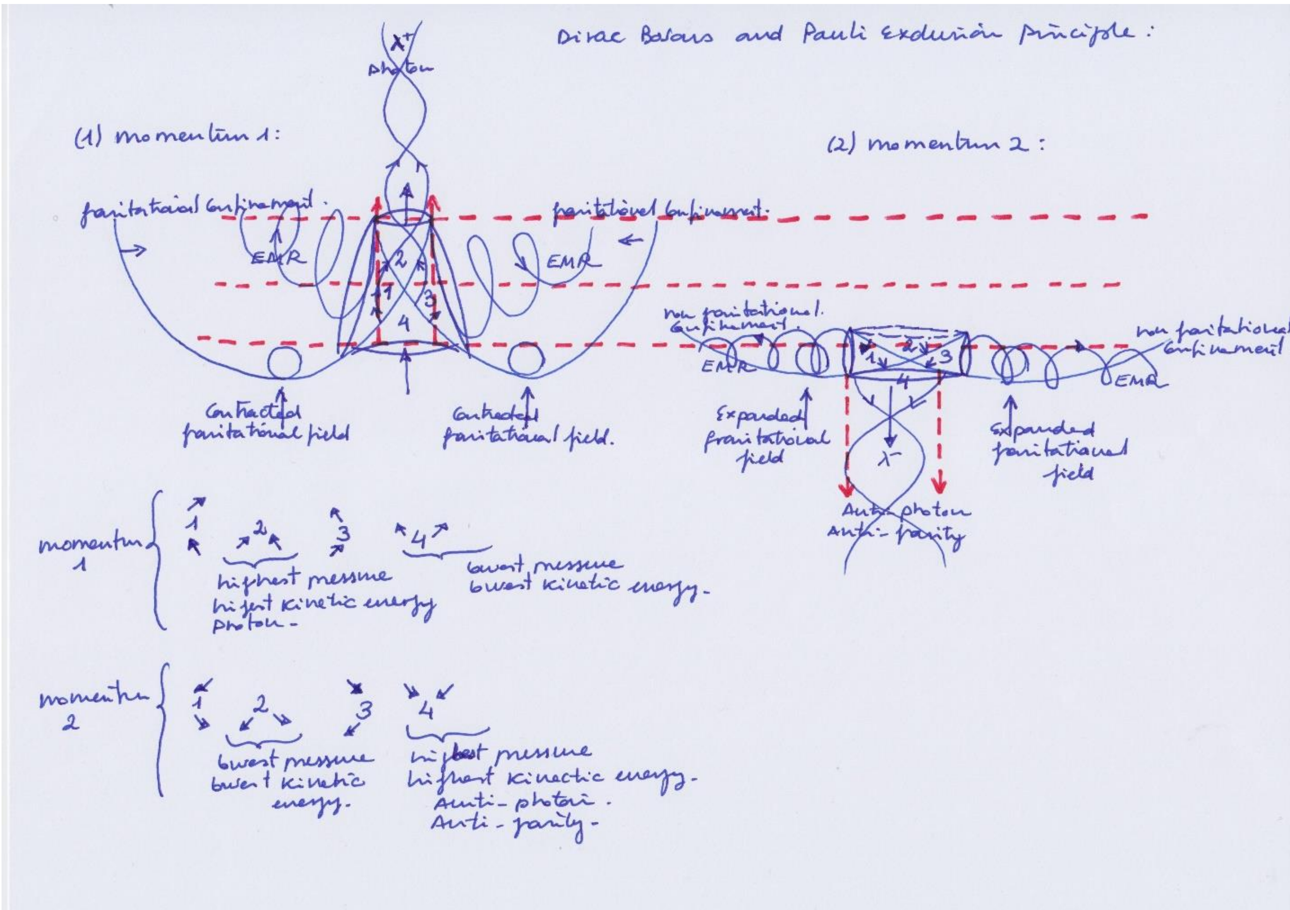

Fig. 28 Symmetric system, (initial diagrams).

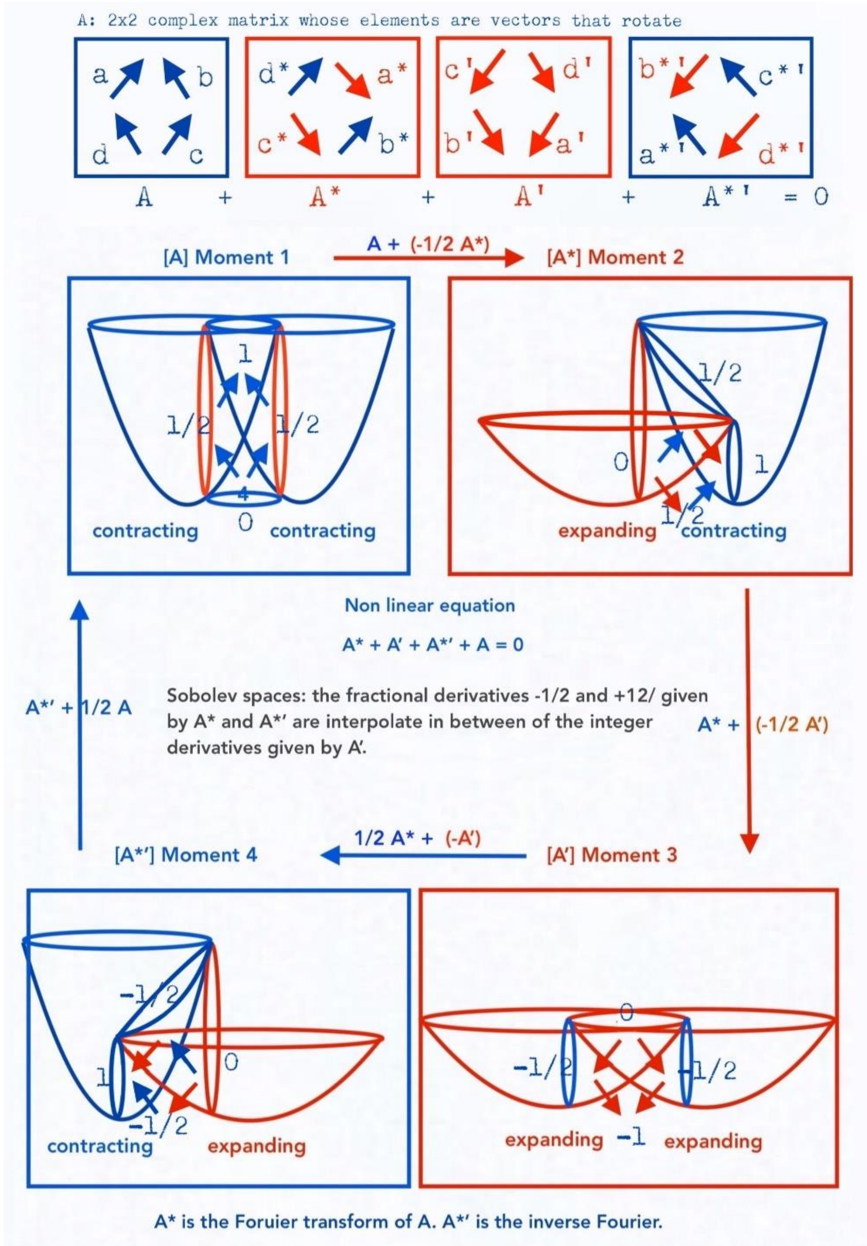

Fig. 29 Fourier transform. Fourier inverse

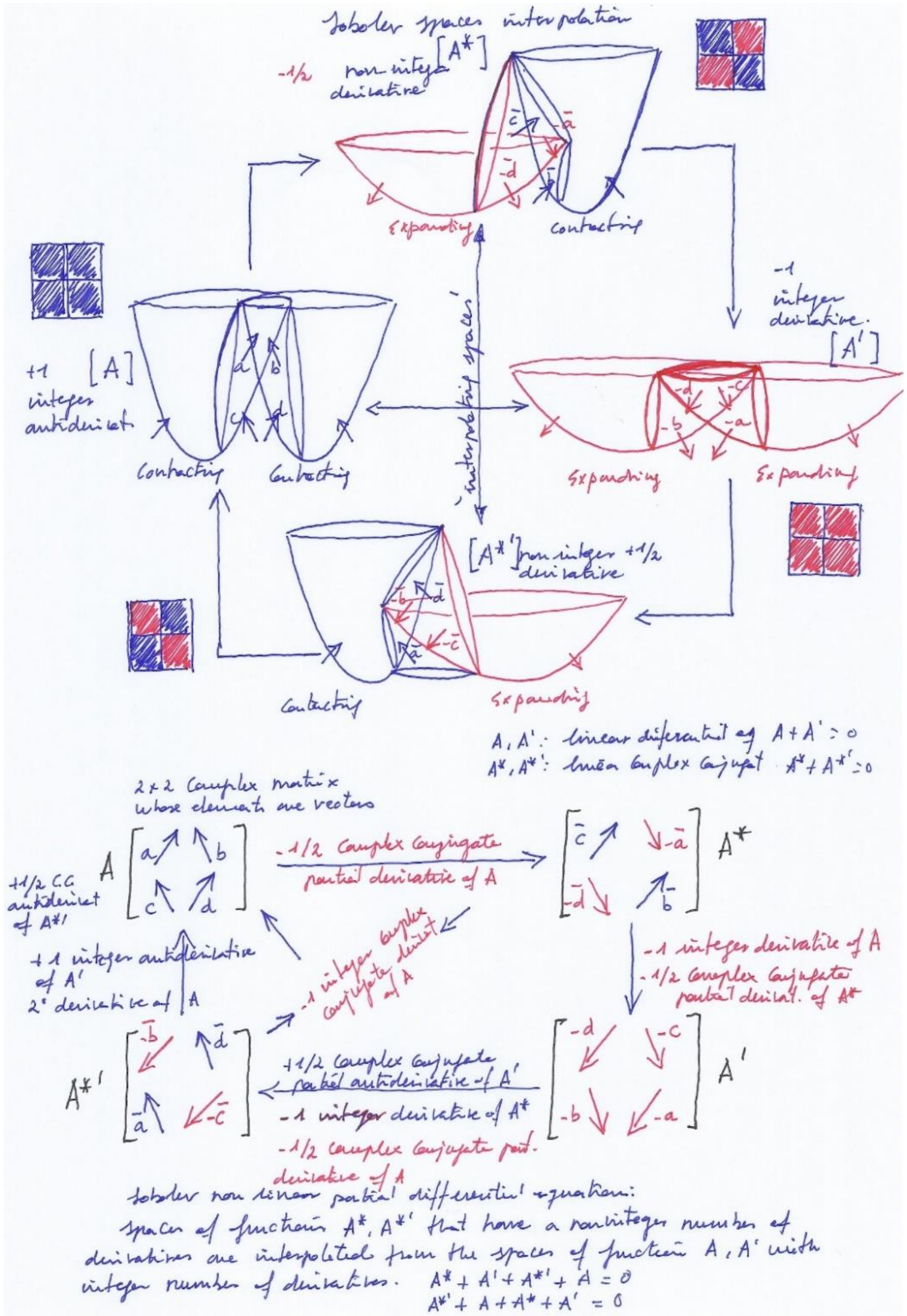

Fig. 30: Modular matrices. Interpolation

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